Macroeconomic models with physical and monetary dimension based on constrained dynamics

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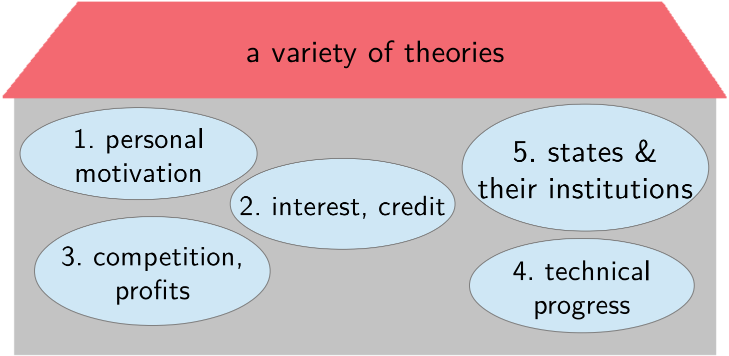
- Motivation: the growth imperative
- Limits of economic models
- Modeling framework
- Model applications: exchange, production, monetary and biophysical stock-flow consistency
- Summary and conclusions

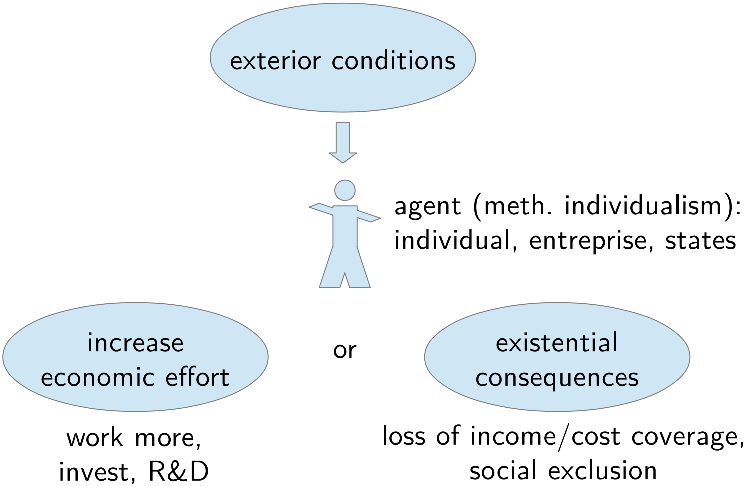
Motivation: Is economic growth imperative?



Survey papers co-authored with Andreas Siemoneit, Berlin:

- Reviewing growth imperatives, Part 1: individuals between desire and social coercion *and* Part 2: businesses, states and the fear of stagnation. Under Review at Ecological Economics.
- Consistency and Stability Analysis of Models of a Monetary Growth Imperative. In: Ecological Economics 136 (June 2017), pp. 114–125.

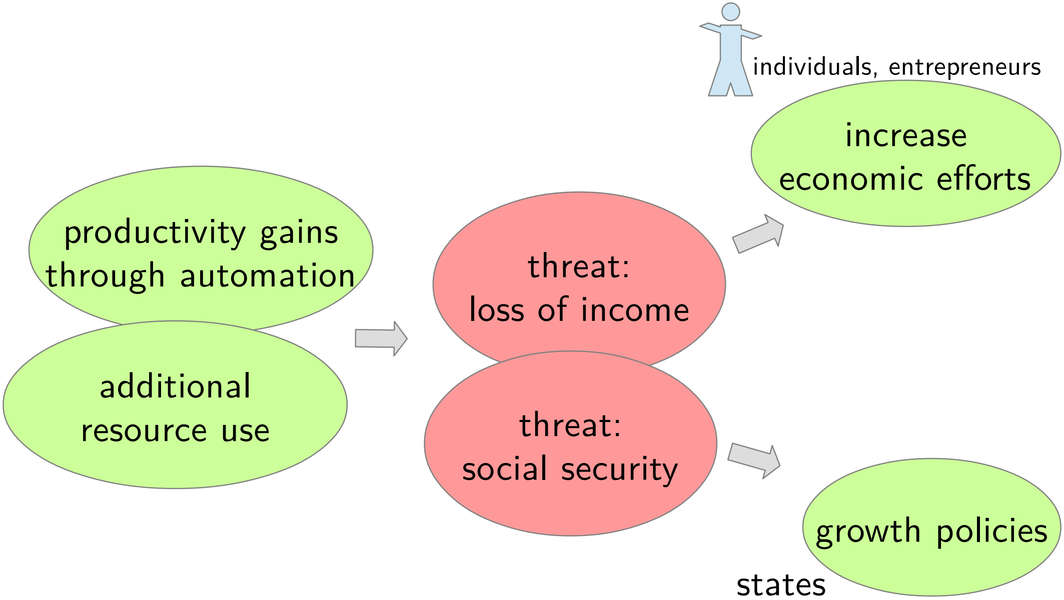
Definition of a growth imperative?



Central role: technical change

- energy use explains Solow residual (Ayres et al., 2009; Kümmel, 2011)
- technical change allows for factor substitution
- vicious and virtuous circle of growth
- without growth: risk of unemployment, indebtedness, instability?

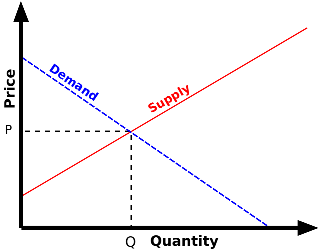
The social dilemma of economic growth



Necessary ingredients of a model

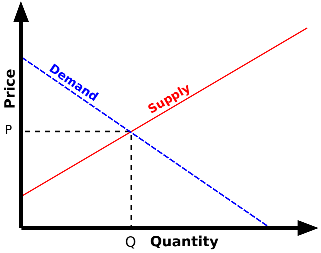
- endogenous resource-driven technical change (Reiner Kümmel, LH14)
- investment, endogenous credit creation (Gaël Giraud, LH14)
- unemployment, income heterogeneity
- conspicuous consumption (no selfish utility functions)
- prisoner's dilemma
- dynamics out of equilibrium

General Equilibrium models



- market clearing
- stochastic shocks, but no uncertainty, instabilities, or coordination failures
- often lack of out-of-equilibrium foundations or multiple equilibria
- no dynamics of money & credit (neutrality)
- rational behavior

BEHAVIORAL ASSUMPTIONS IN GENERAL EQUILIBRIUM MODELS



Constrained optimization of master utility function

- numerous constraints
  (budget, zero-profit equilibrium, ...)
- jump to the utility top
  ("tangent on pareto set"; Yves Bréchet, LH18)
- agents correctly anticipate all constraints
- 'invisible hand' creates no dilemmata
- aggregation has to be possible
  $\rightarrow$ representative agent assumption

- equilibrium models worthless if these conditions do not hold
- how do market forces act out of equilibrium?

Historical analogies between mechanics and economics

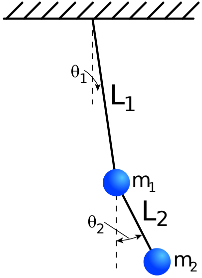
Equilibrium was described in analogy to stationary states of mechanical systems.

Year	Mechanics	Economics
1686, Newton	Dynamics	
1788, Lagrange	Constrained Dynamics	
1838, Cournot		Optimization
1874, Early Neoclassicals / Walras		General Equilibrium
1954, Arrow/Debreu		GE as Optimization under Constraint
2018		Constrained Dynamics?

Role of dynamics in economic models

- Dynamic equilibrium models describe a "quasi-static process": the system is "at equilibrium at every point between its initial and final states" (Berry et al., 1978).
- For early neoclassicals, dynamics "did *not* mean *intertemporal* choices or equilibria but instead the adaptive *processes* that were thought to converge on the states analyzed in *static* theory" (Leijonhufvud, 2006, pp. 29–30)
- "very little has been done to address the unfinished business of the older neoclassical theory" (Leijonhufvud, 2006, pp. 29–30)

General Equilibrium: Inspired by Physics?



Lagrangian Mechanics:

- dynamics of interacting particles under constraints,
- conserved quantities such as energy.

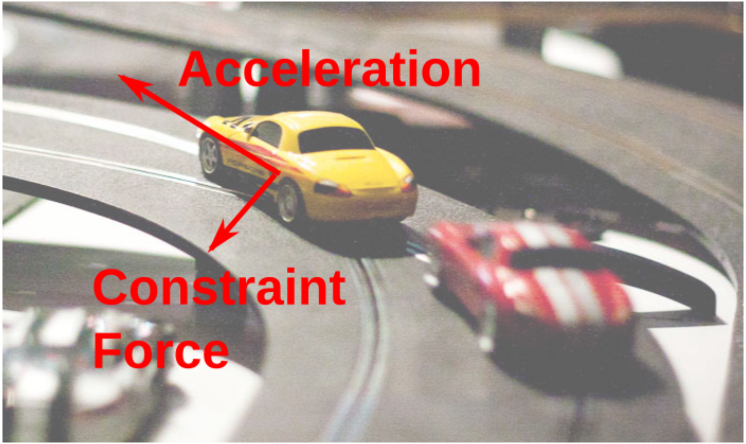
$\Rightarrow$ Newton would have used Stock-Flow Consistent Agent-Based models.

$\Rightarrow$ Idea: Extend analogies between economics and mechanics:

$\Rightarrow$ from constrained optimization to constrained dynamics

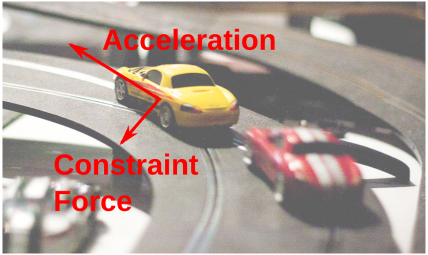
Idea: Motion under Constraint

model dynamics of stocks and flows (goods, financial assets, material, energy) and their restrictions ('consistency')



Relation Lagrangian Mechanics – Economics

Mechanics	**Economics**
velocity v_j	stocks, flows, prices y_j
(constraint) forces f_i^j	(constraint) forces f_i^j
mass M_j	economic power μ_i^j: ability to control a variable
$\dot{v}_j = \frac{1}{M_j} \sum_i f_i^j$	$\dot{y}_j = \sum_i \mu_i^j f_i^j$



Constraints in economic models

Constraints restrict the phase space of the variables:

- individual budget constraints
- production functions (Leontief, Cobb-Douglas, LinEX)
- input–output consistency (Sandra Bouneau, LH18)
- monetary stock-flow consistency (Gaël Giraud, LH14+18): First law of financial economics
- energy conservation: First law of thermodynamics (Reiner Kümmel, LH14)
- mass conservation: First law of chemistry

In our model: Constraints generate constraint forces as in Lagrangian mechanics.

Behavioral assumptions

Optimization / equilibrium models:

- Maximize master utility function under constraints

Our approach:

- different forces as desire to influence certain variables
- economic power is the ability to influence certain variables
- don’t jump to the top, but try to climb the mountain
- allow for heterogeneity among consumers and firms
- no equilibrium assumption, but possible convergence

Differential-algebraic equation framework

m agents
n stocks x_i & corresponding flows $\dot{x}_i$.

Identities / constraints:

$$Z_k(\vec{\dot{x}}) = 0 \tag{1}$$

Time evolution:

$$\ddot{x}_j(t) = \sum_{i=1}^{m} \mu_i^j f_i^j(\vec{x}, \vec{\dot{x}}) + \sum_{k=1}^{l} \lambda_k \frac{\partial Z_k(\vec{x}, \vec{\dot{x}})}{\partial \dot{x}_j} \tag{2}$$

μ_i^j: economic power (ability of agent j to influence flow i)
$f_i^j(\vec{x}, \vec{\dot{x}})$: economic force (wish of i to influence flow j)

(Glötzl, Glötzl, Richters: Discussion Paper 2017)

Four short examples of work in progress

1. Investment and Saving
2. Exchange Model
3. Production Model
4. Monetary Stock-Flow Consistent Model
5. Monetary and Physical Flow Model

Example 2: Exchange Model

Constraints:

$$\sum_i \dot{x}_i = 0 = Z_0. \tag{3}$$

$$\dot{m}_i + p\dot{x}_i = 0 = Z_i \qquad \forall\, i. \tag{4}$$

Gradient climbing: Forces according to marginal utility:

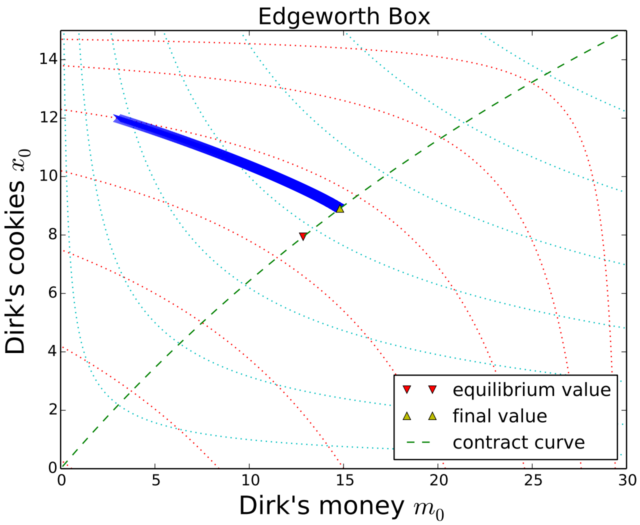
$$f_i^{x_i} \propto \frac{\partial U_i}{\partial x_i}, \qquad f_i^{m_i} \propto \frac{\partial U_i}{\partial m_i}. \tag{5}$$

Time evolution for x_i:

$$\dot{x}_i = \mu_i \frac{\partial U_i}{\partial x_i} + \lambda_0 + p\lambda_i. \tag{6}$$

Slow auctioneer increases price of x slowly if supply > demand.

Edgeworth Box, Exchange Model



Exchange model with ‘slow’ tatonnement process.

Example 3: 2x2 production model

Constraints (for sector i: capital K_i, labor L_i, production C_i):

$$
\begin{aligned}
Z_i &= C_i - K_i^{\kappa_i} L_i^{1-\kappa_i} = 0, \\
Z_K &= \dot{K}_1 + \dot{K}_2 = 0, \\
Z_L &= \dot{L}_1 + \dot{L}_2 = 0.
\end{aligned}
$$

Firms increase profits given by:

$$\Pi_i = p_i C_i - r K_i - w L_i.$$

Household increases utility given by:

$$U = C_1^{\alpha_1} C_2^{\alpha_2}.$$

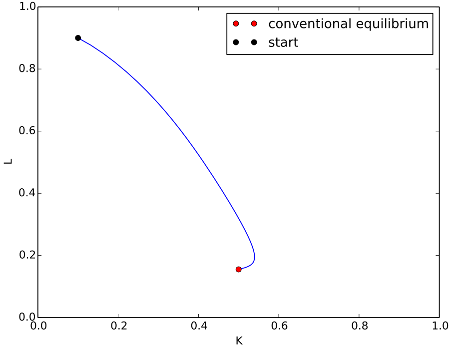
Time evolution (exemplary):

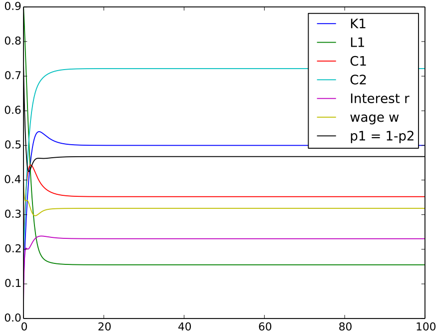
$$\dot{C}_1 = \mu_h \frac{\partial U}{\partial C_1} + \mu_{f1} \frac{\partial \Pi_1}{\partial C_1} + \lambda_i.$$

Slow price adaptation:

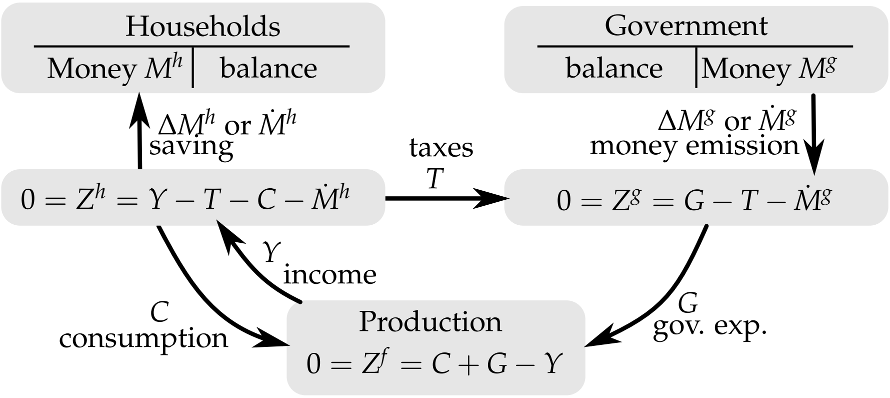
$$\dot{r} = \mu^r \sum_i \dot{K}_i^\top.$$

Time evolution and convergence





Example 4: Stock-Flow Consistent Models



(Godley et al., 2007)

- constraints: consistency of stocks and flows
- behavioral functions and disequilibrium behavior
- discrete dynamical system, motion under constraint

Challenges with SFC models

Problems with these discrete time models

- N variables, together with K constraints
- (arbitrary) subset of $N - K$ behavioral functions can be chosen
- Example: consumption function: $C_{(t)} = c_y Y_{D(t)} + c_v M_{(t-1)}$

Our approach:

- behavioral forces for each variable
- needed: K additional Lagrange multipliers

Rebuild model as Constraint Dynamics with Utility Functions

Household's utility U depends on consumption C and money stock M.

Government spending G is exogenous, θ: tax rate. $Y = C + G$.

Constraint:

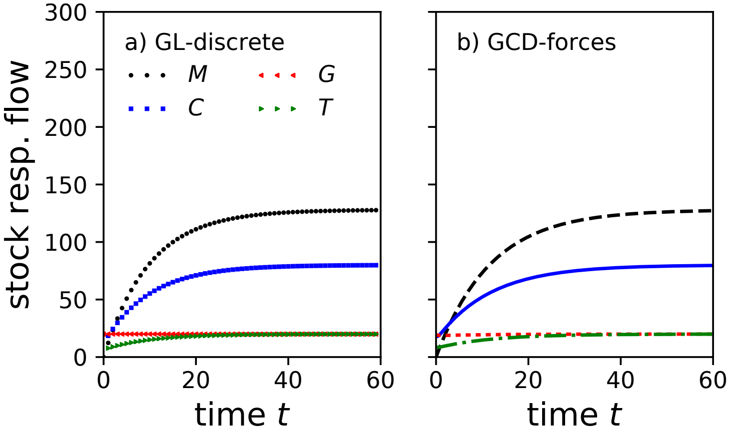
$$Z_0 = (1-\theta)(C+G) - C - \dot{M} = 0. \tag{7}$$

Time evolution:

$$\dot{C} = \mu \frac{\partial U}{\partial C} - \theta\lambda, \tag{8}$$

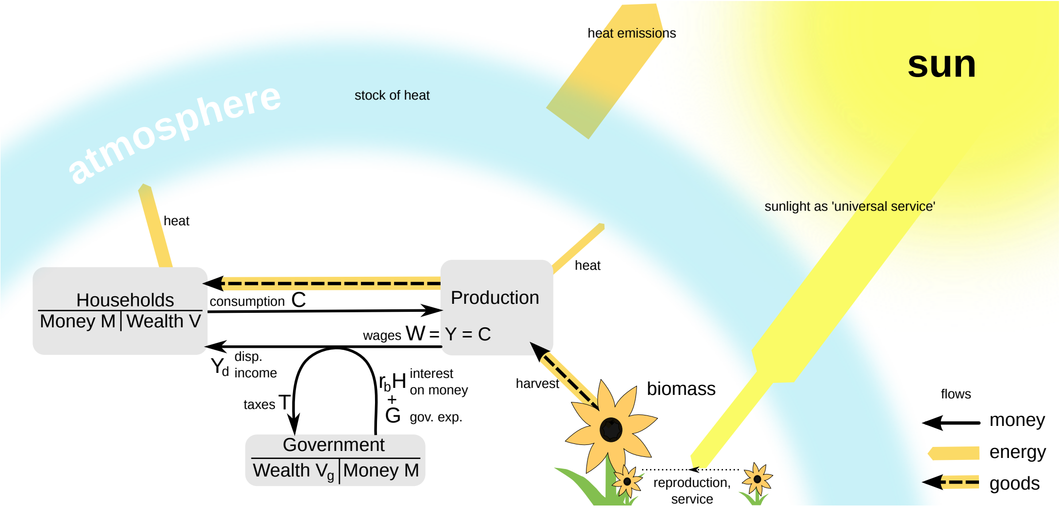
$$\ddot{M} = \mu \frac{\partial U}{\partial M} - \lambda. \tag{9}$$

Result of the reproduction



upper left: discrete original model.
others: continuous imitation with utility gradient climbing

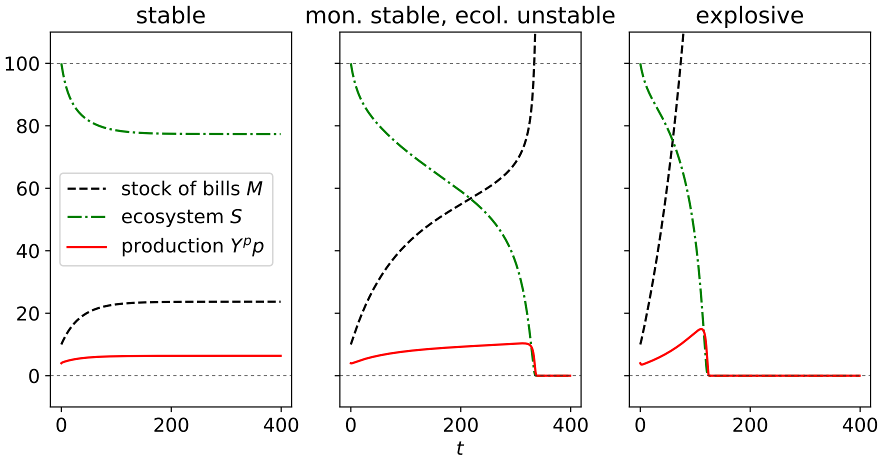
Example 5: Ecological–Financial Model



Ecological–Financial Model

- demand-driven monetary SFC model, including interest-bearing debt
- flows and funds of energy (Georgescu-Roegen, 1971)
- ecosystem exhibits logistic growth

STABILITY ANALYSIS: ECO-ECO-INTERACTION



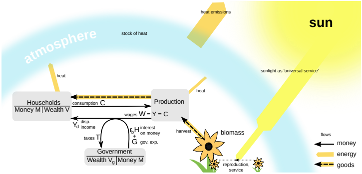
(Barth et al., 2018)

Characteristics of the framework

(1) incorporate behavioral assumptions different from optimization,
(2) relax macroscopic assumptions about aggregation of individual behavior,
(3) distinguish and model of *ex-ante* and *ex-post* dynamics,
(4) discuss slow price adaptation and out-of-equilibrium dynamics,
(5) treat stocks, flows, and their constraints consistently,
(6) formalize economic power, and
(7) include some well-known general equilibrium solutions as fixed points of the dynamical system.

Different economic theories can be represented within one single framework.

CONCLUSION



Work in progress:

- Constrained dynamics formalize economic forces, constraint forces, and power for economic models in and out of equilibrium.
- Models represent goods, production, money, energy, and materials consistently.
- To do: Combine and apply them.

Questions or feedback?

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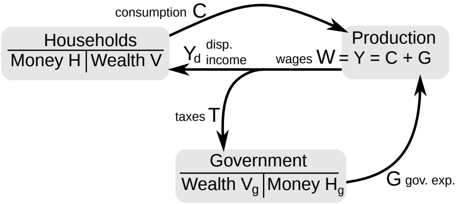
Example 3: Post-Keynesian Stock-Flow Consistent Model



(Godley et al., 2007)

Dynamical equations:

$$C = c_y \cdot Y_D + c_v \cdot H_{(t-1)}, \tag{10}$$

$$Y_D = (1-\theta) W \cdot N, \tag{11}$$

$$T = \theta \cdot W \cdot N, \tag{12}$$

$$\Delta H = G - T = Y_D - C. \tag{13}$$

Y_D disposable income, C consumption, H money stock of households, W wage per hour, N hours worked, T taxes, θ tax rate, G government expenditures, consumption out of income (c_y) and wealth (c_v).

Rebuild model as Constraint Dynamics with Utility Functions

Assume a U_h depends on consumption C and money stock H. Government spending G is exogenous. Constraint:

$$Z_0 = 0 = (1-\theta)(C+G) - C - \dot{H}. \tag{14}$$

Time evolution:

$$\dot{C} = \mu \frac{\partial U}{\partial C} - \theta \lambda_0, \tag{15}$$

$$\ddot{H} = \mu \frac{\partial U}{\partial H} - \lambda_0. \tag{16}$$