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## **On Economics and Energy**

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#### Content

Future Model Differential Forms and Line Integrals Accounting The Laws of Economics Production and Growth Summary

## **Future Model**

Why do we teach our children and students the knowledge of our past, even though they need the tools of tomorrow?

We do this, because many things of today will be valid (V) in the future, but, of course, many things of the future are today still unknown (U):

The future elements may be divided into two categories:

- V: <u>always valid</u> now and in the future known
- U: presently unknown only in the future known

## **Examples of the Future Model**

#### Science

In science all elements may be divided into two categories:

V: <u>always valid</u> – now and in the future valid – Teaching facts

U: <u>presently unknown</u> – in the future known – Research results

## Economics

In economics all elements may be divided into two categories:

V: <u>ex ante</u>: (always known) – like functions

U: <u>ex post:</u> (only in the future known) – like income

(We can file our income tax return only at the end of the year!)

#### **Physics**

Physical systems may be divided into two categories:

- V: <u>conservative</u>: (always known) e.g. motion in conservative forces (Kepler's laws (1619) are still valid today, as space has no friction.)
- U: <u>non-conservatice</u>: (only in the future known) –e.g. turbulent motion

## **Mathematical Examples of the Future Model**

#### Calculus

Two dimensional calculus may be divided into two categories: V: <u>exact</u> differentials (d f, with known stem function f)

U: <u>not-exact</u> differentials ( $\delta$  g, with not existing stem function)

## Integrals

In calculus integrals may be divided into two categories:

- V: <u>Riemann</u> integrals (path independent, known)
- U: Stokes integrals (path dependent, path unknown)

## **Statistics**

In statistics all elements may be divided into two categories:

V: constraints (precise value known)

U: <u>probable</u> (precise value unknown)

## Data

Data are real numbers and may be divided into two categories:

V: rational (solution of a linear equation: x = B/A, solution is known)

U: <u>irrational</u> (solution of a non-linear equation,  $x \neq B/A$ , gen. unknown)

#### **Economics:** Possible mathematical tools:

#### **Differential forms** (Macroeconomics)

V: all <u>ex ante</u> terms must be written as <u>exact differentials</u> U: all <u>ex post</u> terms must be written as <u>not-exact differentials</u>

#### **Line integrals** (Macroeconomics)

V: all <u>ex ante</u> terms must be written as <u>Riemann</u> integrals U: all <u>ex post</u> terms must be written as <u>Stokes</u> integrals

#### **Statistical Theory (Microeconomics)**

V: all <u>ex ante</u> terms must be written as constraints, as functions U: all <u>ex post</u> terms must be written as <u>probability</u> terms

#### **Chaos Theory and Differential Equations (Complexity)**

V: all <u>ex ante</u> terms must be written as linear diff. equations U: all <u>ex post</u> terms must be written as non-linear diff. equations

## **Differential Forms and Line Integrals**

#### Calculus of two variables

## V: Exact differential dF Example

F is the stem function:

 $F = F(x, y) F = x^{4} y^{7}$   $d F = F_{x} d x + F_{y} d y d F = 4 x^{3} y^{7} d x + 7 x^{4} y^{6} d y$   $F_{xy} = F_{yx} 28 x^{3} y^{6} = 28 x^{3} y^{6}$ 

#### <u>U: Not exact differential $\delta$ M</u> There is no stem function M:

There is no stem function M:

 $\delta \mathbf{M} = \mathbf{x} \, \mathbf{d} \, \mathbf{F}$  $\delta \mathbf{M} = 4 \, x^4 \, y^7 \, \mathbf{d} \, \mathbf{x} + 7 \, x^5 \, y^6 \, \mathbf{d} \, \mathbf{y}$  $\mathbf{M}_{\mathbf{x}\mathbf{y}} \neq \mathbf{M}_{\mathbf{y}\mathbf{x}}$  $28 \, x^4 \, y^6 \neq 35 \, x^4 \, y^6$ 

**Example** 

<u>U</u> $\rightarrow$ V: Integrating factor (1/x): d F = (1/x)  $\delta$  M

#### V: A Closed Riemann line integral

#### of an exact differential (d F) is always zero!

Integral along path AB is equal to the integal along path BA (**Equilibrium**)



**Example**: mechanical energy at conservative forces

#### **U: A Closed Stokes line integrals**

of not-exact differentials ( $\delta$  M) is never zero!

Integral along path AB is not equal to integral along path BA (Non-equilibrium)



Example: magnetic field, vortex field: High, Low in weather.

## Accounting

## Neoclassical flow model

A household (H) works in industry (In) earning  $100 \notin$  per day and spending  $90 \notin$  for food and goods. The surplus is  $10 \notin$  per day.



Income flows from industry to households and consumption costs flow back to industy. The surplus flows to the bank and back to industry for investments.

## Neoclassical flow model

A household (H) works in industry (In) earning  $100 \notin$  per day and spending  $90 \notin$  for food and goods. The surplus is  $10 \notin$  per day.



Imagine: Industry pays  $100 \in$  and receives only  $90 \in$  as consumption costs. The next day industry must borrow  $10 \in$  from the bank to pay the income again! This model does not work for industry! **The neoclassical flow model is invalid**!

## An unbalanced account is a closed Stokes line integral

A household (H) works in industry (In) earning  $100 \notin$  per day and spending  $90 \notin$  for food and goods. The surplus is  $10 \notin$  per day.



## Luca Pacioli (1494): Double-entry accounting

A household (H) works in industry (In) earning  $100 \notin$  per day and spending  $90 \notin$  for food and goods. The surplus is  $10 \notin$  per day.

(monetary units:  $\in$ ) (naturally: energy units, kcal, MJ)

<b>Y</b> <sub>H</sub> =	100	€	<b>W</b> <sub>H</sub> =	- <mark>1</mark> 00	€
C <sub>H</sub> =	-90	€	<b>G</b> <sub>H</sub> =	90	€
$\Delta M_{\rm H} =$	10	€	$\Delta P_{\rm H} =$	- <b>10</b>	€

monetary account + productive account = 0

The monetary account measures the productive account (in €)

## Double-entry accounting in Stokes integrals

A household (H) works in industry (In) earning  $100 \notin$  per day and spending  $90 \notin$  for food and goods. The surplus is  $10 \notin$  per day.

(monetary units:  $\epsilon$ ) (naturally: energy units, kcal, MJ)

$$\oint \delta M + \oint \delta L = \mathbf{0}$$

The monetary account measures the productive account (in  $\in$ )

Double-entry accounting – the basis of economics for 500 years

## **The Laws of Economics**

## **Econophysics: The laws of Macro-Economics**

$$\oint \delta \mathbf{M} = -\oint \delta \mathbf{L}$$

Pacioli law in integrals

**1. law:**  $\delta M = d K - \delta L$  Differential balance *Output = Capital Labour* 

**2. law:**  $\delta \mathbf{M} = \lambda \mathbf{d} \mathbf{F}$ 

Existence proof of the production function F

## **Econophysics: The laws of Macro-Economics**

$$\oint \delta \mathbf{M} = -\oint \delta \mathbf{L}$$

Pacioli law

**1. law:**  $\delta M = d K - \delta L$  Differential balance Output = Capital Labour

**2. law:**  $\delta M = \lambda d F$  Existence proof of the production function F

The 2. law replaces the neoclassical Solow equation;  $\delta Y = \lambda d F$  instead Y = A F (K, N)The Solow model is valid only for constant  $\lambda = A$ . The fundamental laws of Economics

 $\underline{\text{monetary units}} \qquad \underline{\text{energy units}}$   $\oint \delta M = -\oint \delta L \qquad \oint \delta Q = -\oint \delta W$ 

1. law:  $\delta M = d K - \delta L$ 

 $\delta \mathbf{Q} = \mathbf{d} \mathbf{E} - \delta \mathbf{W}$ 

2. law:  $\delta M = \lambda d F$   $\delta Q = T d S$ 

## The structural identity of economics and thermodynamics

Symbol	Economics	Unit		Symbol	Thermodynamics	Unit
М	Profit, loss	€, \$, £. ¥	$\leftrightarrow$	Q	Heat	kcal, kWh
K	Capital	€, \$, £. ¥	$\leftrightarrow$	Е	Energy	kcal, kWh
W	Wealth	€, \$, £. ¥	$\leftrightarrow$	Н	Enthalpy	kcal, kWh
Р	Production, Labour	€, \$, £. ¥	$\leftrightarrow$	W	Work	kcal, kWh
λ	Mean capital	€, \$, £. ¥	$\leftrightarrow$	Τ	Mean energy	kcal, kWh
F	Production function	-	$\leftrightarrow$	S	Entropy	-
Ν	Number	-	$\leftrightarrow$	Ν	Number	-
р	Price per item	€, \$, £. ¥	$\leftrightarrow$	р	Pressure	kcal / m³
V	Volume, amount	-	$\leftrightarrow$	V	Volume	m <sup>3</sup>
L	Lagrange function	€, \$, £. ¥	$\leftrightarrow$	F	Free energy	kcal, kWh
L*	LeChatelier function	€, \$, £. ¥	$\leftrightarrow$	G	Free Enthalpy	kcal, kWh

Table I Corresponding terms in economics and thermodynamics

The temperature ( $\lambda$ ) of an economic/social system:



GDP per capita and energy consumption per capita

## **Production Function and Entropy**

Entropy replaces the Cobb-Douglas production function

#### **S : Shannon Entropy**

(Georgescu-Roegen)

#### **F : Cobb Douglas**

(binary system)



Entropy S = ln p is the link between macro- and microeconomics

## **Entropy and Disorder**

 $\delta W = d E - T d S$ 

A light breeze (E) in a park will easily empty a paper basket and generate disorder (S). The paper will never come back into the basket.

But a janitor may work and sweep the paper together and put it back into the basket. Work (W) reduces entropy: Work is ordering!



## **Entropy and Work/Production/Labour**

 $\delta L = d K - \lambda d F$ 

Production ( $\delta$  L) is ordering, entropy reduction (- d F)

Production: 
$$\bigcirc + \bigcirc + \bigcirc + \bigcirc \rightarrow \bigcirc \bigcirc$$

#### Brain work: $g+i+r+n+o+d+e \rightarrow$ ordering

## Production costs ( $\delta$ L) depend on the standard of living ( $\lambda$ ) of a country

## **Production and Growth**

## **<u>Carnot cycle</u>** in motors and production circuits

$$\oint \delta Q = \oint T \, \mathrm{d} S$$

$$\oint \delta M = \oint \lambda \, \mathrm{d} \, \mathrm{F}$$





Motor

Production circuit

The mechanism of production (Carnot process)

$$\oint \delta M = \oint \lambda \, \mathrm{d} \, \mathrm{F}$$



λ

Production of coal in SA and trade with EU

F

#### Economic growth: Distribution of profits

$$d \lambda_{1}(t) = p (\lambda_{2} - \lambda_{1}) d \alpha t$$
$$d \lambda_{2}(t) = (1 - p) (\lambda_{2} - \lambda_{1}) d \alpha t$$

**p** : distribution to lower side



Two interdependent countries



Growth: 0

Stagnation: p = 0,75

#### Economic growth by international trade

## Growth by international trade:



US - China 1990-2010

#### Economic growth by trade

## Growth in the world: Labour: 3 % Capital 6 %



#### **Data by Thomas Piketty**

## Wealth: combining Capital and Energy

<u>Economics</u>	<b>Thermodynamics</b>
Wealth: $W = K + P V$	H = E + p V
1. law: $\delta M = d W - V d P$	$\delta Q = d H - V d p$
2. law: $\delta M = \lambda d F$	$\delta \mathbf{Q} = \mathbf{T} \mathbf{d} \mathbf{S}$
$L^*(P, \lambda) = W - \lambda d F \rightarrow max!$	$G = H - T d S \rightarrow max!$

## Conclusion

The new approach to economics by calculus, stochastic theory or chaos theory leads to results, that differ considerably from mainstream economics.

From the approach by calculus we learn:

1. The laws of macro-economics must be written in differential or integral form.

The Solow model of neoclassical economics must be replaced by the second law, δ M = λ d F. The integrating factor λ is the standard of living, the GDP per capita or the energy consumption per capita.
 The Cobb Douglas production function F must be replaced by the Shannon entropy.

4. Business is not a cycle, but a spiral that goes up (growth) or down (deficit).

5. Production is a two level cyclic Carnot process: buying cheap and selling expensive, like hot and cold in a motor, EROI >>1.

6. The production factors capital and labour correspond to energy and work. Economic laws may be given either in monetary or in energy terms. Mixing capital and energy is not  $Y \neq F(K, E, L)$  but W = K + PV.

## Thank you for your attention

## EROI, economic and political state

#### **<u>Two level industrial production:</u>** EROI $\varepsilon \sim (\lambda_2 / \lambda_1)$

- a) Capitalism: High surplus is divided between rich and poor, 90 : 10  $\varepsilon = 9$ : High prices, low wages, high surplus, strong market, low unemployment, high economic efficiency, *exponential* growth Example: *Industrial countries (USA, Germany)*
- **b)** Socialism: The surplus is divided between rich and poor,  $\varepsilon = 75:25$  $\varepsilon = 3$ : Lower prices, higher wages, less surplus, less strong market higher unemployment, less exponential growth Example: *Industrial countries*, (France)
- c) Rural production: Without industrial production the surplus is small.  $\varepsilon \rightarrow 1$ : Low prices, low wages, weak markets, slow economic growth, rural seasonal employment, low efficiency *Example: Rural countries, (many African countries: electrify Africa!)*
- **d) Communism:**  $\lambda_2 = \lambda_1$ : capital is included in the proletarian class.  $\varepsilon = 1$ : Low prices, low wages, markets crash. no growth, no unemployment Example: *Communist countries (former USSR block countries)*

May we mix capital and electomagnetic energy?

Y = F(K, E, L)? No!

Thermodynamics introduces electomagnetic fields D, E, H, B by  $\delta W = (-B d H - E d D) V$ 

 $\delta Q = d E + p d V + (D d E + H d M) V$ 

If we assume a corresponding terms in economics, we obtain

 $\delta Y = d K + P d V + (D d E + H d M) V$ 

But so far the meaning of D, E, H, M in economics is unknown!

#### **Economics: Calculus, Stochastic Theory, Chaos Theory**

#### Calculus

- 1. 1. and 2. law of economics
- 2. Production and trade
- 3. Economic growth

#### **Stochasic Theory**

- 1. Heterogeneous agents
- 2. Financial markets, financial crisis
- 3. Networks
- 4. Game theory, cooperation, competition

#### **Chaos Theory and non-linear differential equations**

- 1. Order disorder
- 2. Turbulence, chaos
- 3. Fractals

#### **Outlook: Calculus based Economics and Social Sciences**

#### Economics

- 1. Production and trade
- 2. Economic growth
- 3. Income distributions; How much should a manager earn?
- 4. Financial markets, financial crisis

#### Societies

- 1. Homogeneous societies: state, army, church, companies
- 2. Phases: collective, individual, global, transitions: revolutions, crisis
- 3. Heterogeneous societies: cooperation, integration, segregation, aggression: Example: The war in Bosnia
- 4. Public decisions: The analysis of US elections
- 5. Marriage analysis: Mobility in Germany

#### **Politics**

- 1. Standard of living and political system; capitalism, socialism, communism
- 2. Oil and the Arab revolution
- 3. E U: From Marshall plan to integration and present problems: Greece
- 4. Migration and refugies in EU countries

## **Financial markets**

#### Look back into the Future





The fundamental laws of Economics

 $\underline{\text{monetary units}} \qquad \underline{\text{energy units}}$   $\oint \delta M = -\oint \delta L \qquad \oint \delta Q = -\oint \delta W$ 

1. law:  $\delta M = d K - \delta L$ 

 $\delta \mathbf{Q} = \mathbf{d} \mathbf{E} - \delta \mathbf{W}$ 

**2. law:**  $\delta \mathbf{M} = \lambda \mathbf{d} \mathbf{F}$ 

 $\delta \mathbf{Q} = \mathbf{T} \mathbf{d} \mathbf{S}$ 

Oilprice:



## Look back into the past: from energy to money





#### Natural production circuit

Food: kcal Work: MJ Capital: Fields

#### Modern production circuit

Income: €, \$ Labour: €, \$ Capital: Industry

## The dynamics of industrial production

Production is a two level (Carnot) process with  $\lambda_2$ ,  $\lambda_1$ 

**Efficiency:**  $\eta \sim (\lambda_2 - \lambda_1) \rightarrow \max!$ 

A running motor gets hotter, the efficiency grows with time! A running economy gets richer, the efficiency, the difference between capital and labour grows with time!



The growing gap between rich and poor is the result of the dynamics of production. (Carnot process)

#### What determines the rising gap between rich and poor?

The distribution of annual surplus between the levels  $\lambda_2$  and  $\lambda_{1,}$  between rich and poor, between capital and labour.

**Application: Finance** 

Banks invest in financial (d K) and productive ( $\delta$  L ) markets.

$$\oint \delta Q = \oint dK - \oint \delta L > 0!$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

$$= 0 \qquad \neq 0 \qquad \downarrow$$

Only investments in labour/production ( $\delta$  L) lead to economic growth

Capital (d K) by itself does not create capital! Investing only in financial markets (d K) is like gambling or playing roulette

$$\oint \delta M = \oint dK - \oint \delta L$$
$$= 0 + 7,7\%$$



a) <u>-  $\delta$  L: Long time</u> returns lead to growth in productive (stock) markets (7,7 % p. a for 60 y)

b) <u>d K</u>: <u>Short time</u> returns lead to symmetric volatility – no growth - in stock markets

# Strategies in high risk finance $\oint dK = 0!$



**Creation of bubbles** 

## Results

**Double entry accounting:** 
$$\oint \delta M = -\oint \delta L$$

Macroeconomics:

1. law: 
$$\delta M = d K - \delta L$$
  
2. law:  $\delta M = \lambda d F$   
Trade, Production:  $\Delta M = (\lambda_2 - \lambda_1) \Delta F$ 

Lagrange:  $L = K - \lambda F$ 

Microeconomics:Lagrange: $L = K - \lambda \ln p \rightarrow \min!$ Shannon entropy: $F = \ln p = -N \{x \ln x + y \ln y\} = -N \ln \{x \ ^x y \ ^y\}$ Financial systems: $\oint d K = 0$ 

Production and finance:	- L = prod. market + $\lambda$ financ. market $\rightarrow$ max!					
Probability and constraints:	- L =	constrai	nts + $\lambda$ probability	$\rightarrow$ max!		
Systems: Social, Politics	- L =	order	$+\lambda$ disorder	$\rightarrow$ max!		

## <u>Lagrange function of systems:</u> $L = V + \lambda U$

Lagrange:	$\Gamma =$	E	+	$\lambda \ln P$	$\rightarrow$ maximum!
Probability:	L =	constraint	+	$\lambda$ probability	$\rightarrow$ maximum!
Thermodynamics:	L =	enthalpy	+	$\lambda$ entropy	$\rightarrow$ maximum!
Laws:	L =	order	+	$\lambda$ disorder	$\rightarrow$ maximum!
System:	$\Gamma =$	perfect	+	$\lambda$ imperfect	$\rightarrow$ maximum!
Reality:	L =	ideal	+	λ real	$\rightarrow$ maximum!
Complexity:	$\Gamma =$	simple	+	$\lambda$ complex	$\rightarrow$ maximum!
Appearance:	L =	uniform	+	$\lambda$ variety	$\rightarrow$ maximum!
Society:	$\Gamma =$	collective	+	$\lambda$ individual	$\rightarrow$ maximum!
Behavior:	L =	planned	+	$\lambda$ spontaneous	$\rightarrow$ maximum!
Games:	L =	strategy	+	$\lambda$ defect	$\rightarrow$ maximum!
Welfare:	$\Gamma =$	fair	+	λ unfair	$\rightarrow$ maximum!
Common Law:	L =	right	+	$\lambda$ wrong	$\rightarrow$ maximum!
Art:	L =	beautiful	+	λ ugly	$\rightarrow$ maximum!
Music:	L =	harmony	+	$\lambda$ disharmony	$\rightarrow$ maximum!
Health:	L =	healthy	+	$\lambda$ sick	$\rightarrow$ maximum!
Politics:	L =	hierarchy	+	$\lambda$ democracy	$\rightarrow$ maximum
Economy:	L =	capital	+	$\lambda$ chance	$\rightarrow$ maximum!

#### **Weather**

<u>Identification of elements U and V:</u>  $U \neq V$ U: fluctuations

V: conservative variables T, p

Weather predictions are most difficult and can be done only at stable weather conditions with low fluctuations  $u \rightarrow 0$ :  $(u \rightarrow 0) \rightarrow (U \rightarrow 0)$  $W(0, V) \rightarrow W(0, V)$ 

Fluctuations require elaborate computer simulations



Financial state of Europe

#### The Stokes integrals lead to

#### **Economics (capitalism)** with two **separate** levels $\lambda_1$ and $\lambda_2$ :

<b>Companies:</b>	capital and labor investors and savers		
Banks:			
Societies:	rich and poor		
<b>Economies</b> :	1. world and 2. world		

#### <u>**Carnot process**</u> with two **separate** temperatures $T_1$ and $T_2$ :

Motors	hot and cold
Refrigerators	hot and cold
Heat pumps	hot and cold
Generators	hot and cold
Kerrigerators Heat pumps Generators	hot and colo hot and colo hot and colo

#### **Econophysics**:

## The Carnot process of companies and motors

$$\oint \delta M = \oint \lambda \, d \, F = - \oint \delta P$$



Capitalism and the Carnot cycle of motors, both run on the same fuel: oil.

## **Entropy: the natural production function**

## <u>Shannon</u>: $d S = -d N \Sigma (p_i) \ln (p_i) \ge 0$



If we open a perfume bottle, the perfume will leave the bottle to occupy a more probable state and it will never return into a less probable state in the bottle.

## **Examples of model equations:**

#### Thermodynamics

The first law of thermodynamics:

 $\delta Q = d E - \delta W$ 

V: the <u>conservative</u> term *energy* is written as <u>exact differential</u>: d E

U: the <u>not-conservative</u> terms *heat* and *work* are written as <u>not-exact differentials</u>:  $\delta Q$ ,  $\delta W$ .

#### **Neoclassical Macroeconomics**

The Solow model:

 $Y = F(\lambda, K, L) = \lambda F(K, L)$ 

- V: the <u>ex ante</u> term *production function* is <u>**not**</u> written as <u>exact differential</u>: d F
- U: the <u>ex post</u> term income is <u>not</u> written as <u>not-exact differential</u>:  $\delta$  Y

Neoclassical theory does not follow the rules of the future model

## Double-entry accounting: Luca Pacioli (1494)

A household (H) works in industry (In) earning  $100 \notin$  per day and spending  $90 \notin$  for food and goods. The surplus is  $10 \notin$  per day.



Double-entry accounting – the basis of economics for 500 years

## **Neoclassical Macroeconomics**

The Solow model:

 $Y = F(\lambda, K, L) = \lambda F(K, L)$ 

Y: output, income (ex post)

F: production function (ex ante)

K: capital (ex ante)

- L: labour (ex post)
- $\lambda$ : factor of technology
- V: the <u>ex ante</u> term *production function* is **not** written as <u>exact differential</u>: d F
- U: the <u>ex post</u> term income is **not** written as <u>not-exact differential</u>:  $\delta Y$

The Solow model does not follow the rules of the future model

Y: only ex post known, cannot be equal to F: ex ante known:  $Y \neq F$ *The future model contradicts the Solow model* 

## The dynamics of capitalism in the world



#### **GdP per capita and fertility for 90 countries**

The fundamental laws of Economics

monetary units energy units  $\oint \delta M = - \oint \delta L$  $\oint \delta Q = - \oint \delta W$  $\delta \mathbf{Q} = \mathbf{d} \mathbf{E} - \delta \mathbf{W}$ 1. law:  $\delta M = d K - \delta L$ 2. law:  $\delta M = \lambda d F$  $\delta \mathbf{O} = \mathbf{T} \mathbf{d} \mathbf{S}$ 

## Oil price: P = K / E = [US \$ / kWh]