What Thermodynamics can teach us about Economy : *theory and practice*

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Introduction

- General historical presentation of thermodynamics paradigm in economy
  - Economic system as thermostatic system
  - Economic system as a self-organized structure operating very far from equilibrium
  - Entropic approach of transactions
- Alternative route to an ecological modelling
  - Energy and matter (E&M)
  - Quality and quantity
  - Time dependent (Intensity and friction)

Economics and thermodynamics : a long term story

- Economics **is** mechanics
  - Scarcity, intensity of the last satisfied need, celestial mechanics

    Walras (1909)
- Economics **is** thermodynamics (at equilibrium)
  - Gibbs formulation

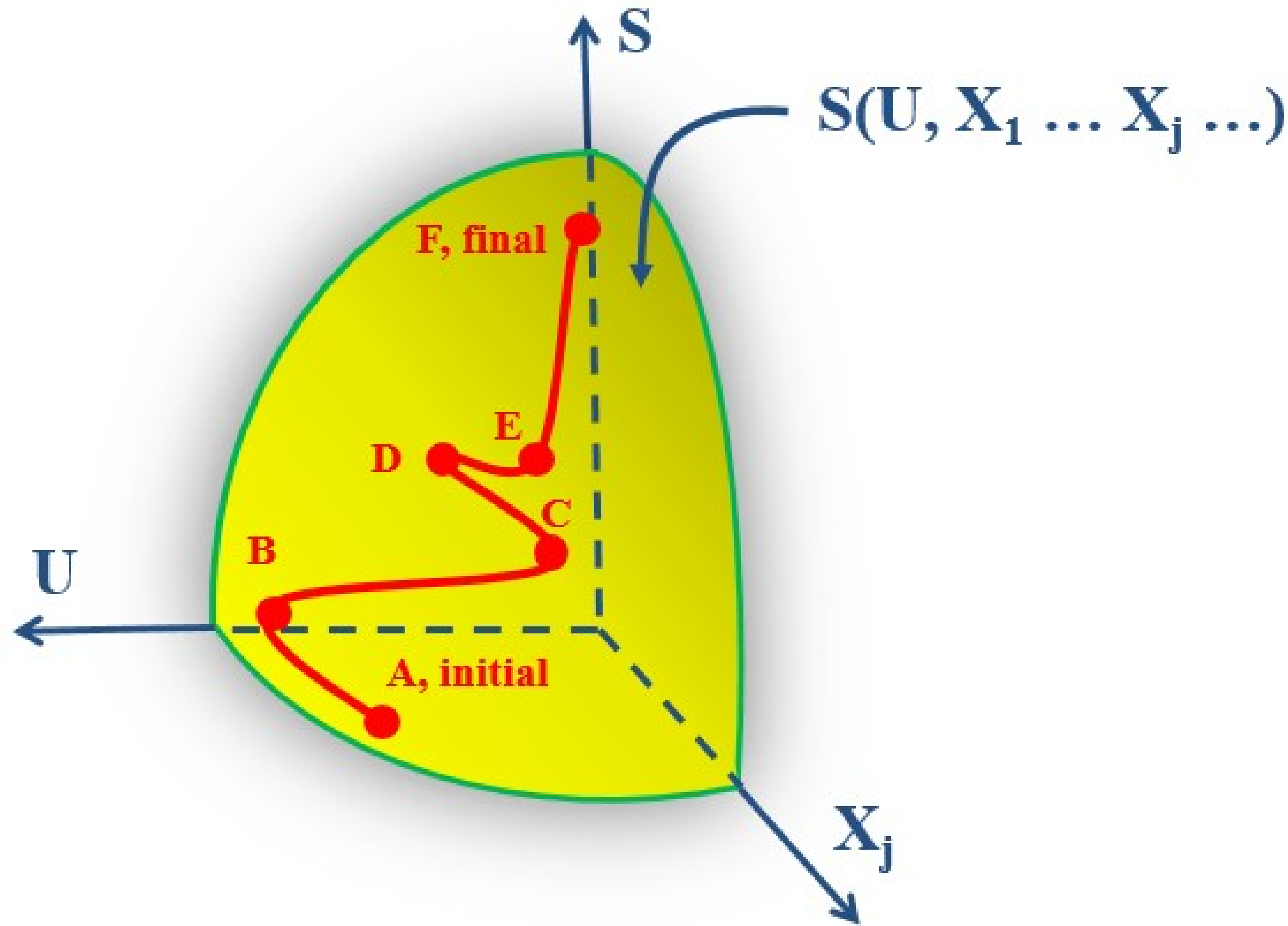
    Soddy (1911) Samuelson (1947)
- Economics **is not** thermodynamics
  - “ Why should there be laws like first and second laws of thermodynamics holding in the economic realm ? ”

    Samuelson (again, but in 1960)
- Economics **is** (again) thermodynamics (out of equilibrium)
  - “ the economic system must strive to reestablish equilibrium whenever it has been disturbed ”

    Schumpeter (1939)
  - “ Matter matters too ”

    Georgescu-Roegen (1971)

General view: Callen's approach



General structure

Adapted from Sousa et al. Physica A, 2006

- Let be an extensive function of state $Y(X_1,...X_N)$
- Let be $z = 1, ... m$ constraints expression
- Y is a function of the N variables X_i , which are extensive quantities

$$L = Y + \sum_z \lambda_z G_z$$

At equilibrium, $\left(\frac{\partial L}{\partial X_i}\right) = 0$ and $\left(\frac{\partial L}{\partial \lambda_z}\right) = 0$ Or $\pi_i = -\sum_z \lambda_z \frac{\partial G_z}{\partial X_i} = \frac{\partial Y}{\partial X_i}$

and $\pi_i(X_1, ...X_N)$ The equations of state

From a thermodynamics point of view

- The Y function is the entropy S = S(U,V,N)
- The X_i variables are U, V, N…
- The constraints $G_z = 0$ are the equations of state and initial/boundary conditions.
- At equilibrium there is no gradient of any intensive potential then $\nabla \pi_i = 0$

$$\pi_U = \left(\frac{\partial S}{\partial U}\right) = \frac{1}{T}$$

$$\pi_V = \left(\frac{\partial S}{\partial V}\right) = \frac{P}{T}$$

$$\pi_N = \left(\frac{\partial S}{\partial N}\right) = -\frac{\mu}{T}$$

From a thermodynamics point of view

- We obtain the Euler fundamental equation

$$dS = \left(\frac{\partial S}{\partial U}\right)_{V,N} dU + \left(\frac{\partial S}{\partial V}\right)_{U,N} dV - \left(\frac{\partial S}{\partial N}\right)_{U,V} dN$$

$$S(U,V,n) = \frac{1}{T}U + \frac{P}{T}V - \frac{\mu}{T}N$$

- Which needs the equations of state to be completed
- Perfect mono-atomic gas:

$$\left|\begin{array}{l} \dfrac{1}{T} = \dfrac{3}{2}R\dfrac{n}{U} \\[2ex] \dfrac{P}{T} = R\dfrac{n}{V} \\[2ex] \dfrac{\mu}{T} - \left(\dfrac{\mu}{T}\right)_0 = -\dfrac{3}{2}R\ln\dfrac{u}{u_0} - \ln\dfrac{v}{v_0} \end{array}\right.$$

From an economics point view

- The Y function is the utility $U_t(X_1, \ldots, X_n)$
- The X_i functions are the (extensives) purchased goods
- The prices density are purchased goods π_i
- They define the (intensive) marginal utilities $\pi_i = \dfrac{\partial Ut}{\partial X_i}$

- Equilibrium: $\nabla \pi_i = 0$

From an economics point of view

- We obtain the Euler fundamental equation

$$dUt = \left(\frac{\partial Ut}{\partial X_1}\right) dX_1 + \left(\frac{\partial Ut}{\partial X_2}\right) dX_2 + ... + \left(\frac{\partial Ut}{\partial N}\right) dX_N$$

$$Ut = \left(\frac{\partial Ut}{\partial X_1}\right) X_1 + \left(\frac{\partial Ut}{\partial X_2}\right) X_2 + ... + \left(\frac{\partial Ut}{\partial N}\right) X_N$$

- Which needs the equations of state to be completed ! (integrability)

Do the marginal utilities π_i have the same statut of potentials, as 1/T, P/T or µ/T have ?

Thermoelastic coefficients, elasticities...

The thermoelastic coefficients are properties of the thermodynamics "material" (fluid, condensed matter...):

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_{P,N} \qquad C_V = T\left(\frac{\partial S}{\partial T}\right)_{V,N}$$

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T,N} \qquad \alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{P,N}$$

Economic elasticities follow from the same derivation: $\varepsilon_{ij} = \frac{\pi_j}{X_i}\left(\frac{\partial X_i}{\partial \pi_j}\right)_{X,\Pi}$

IF the state equations are known $\pi_i = \pi_i(X_1, X_2 ... X_n)$, so does the fundamental equation...then it becomes integrable. (Laurent is still there!)

- Integrability and fundamental function
- Maxwell relations
- Lechatelier-Braun theorem.

Is there an Economic Material ?

Yakovenko (Money temperature)

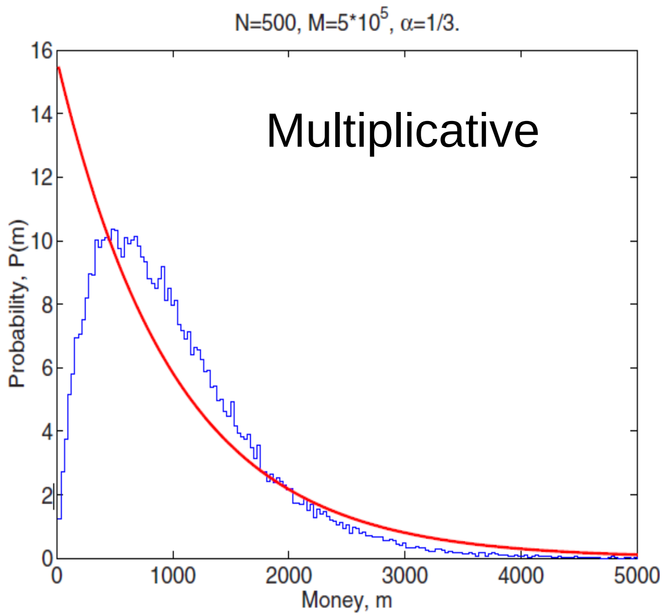
Let the total quantity of money M be extensive
and conservative ditributed over N agents

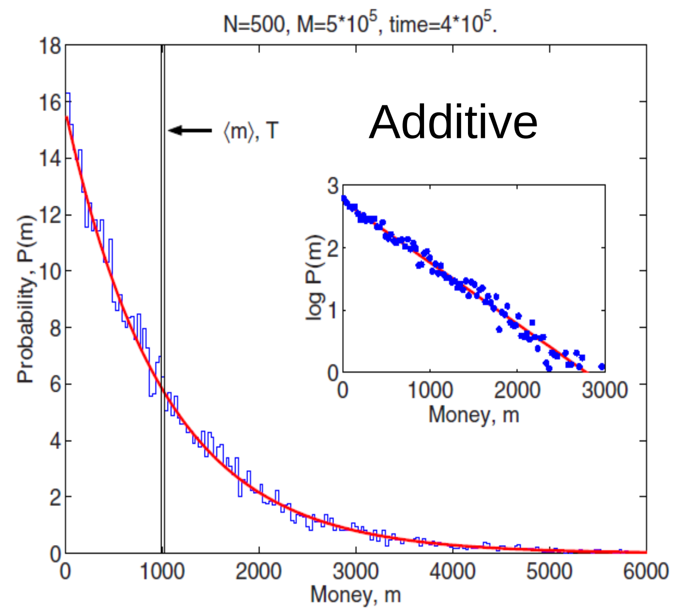
$$N = \sum_i N_i \qquad M = \sum_i N_i m_i$$

(as a Temperature :) $T =< m >= \frac{1}{M} \sum_i p_i m_i \ \frac{M}{N}$

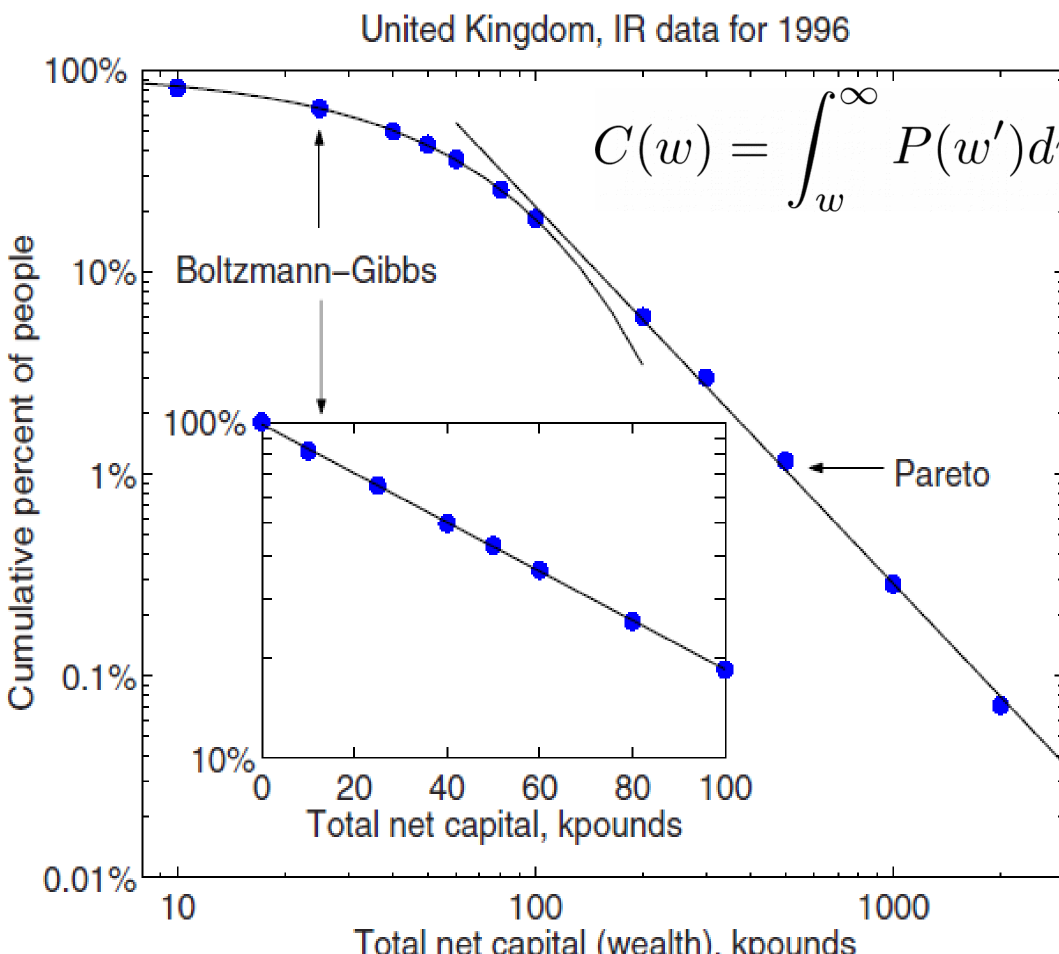
If symmetric exchanges :
Boltzmann-Gibbs distribution

$$p(m_i) = \frac{N_i}{N} = c\exp(-\frac{m_i}{T})$$





Yakovenko (Money temperature)



$$C(w) = \int_w^\infty P(w')dw'$$

There is no problem with such a definition, but why should we call it temperature and not simply the average value of the money carried by agents?

AND!

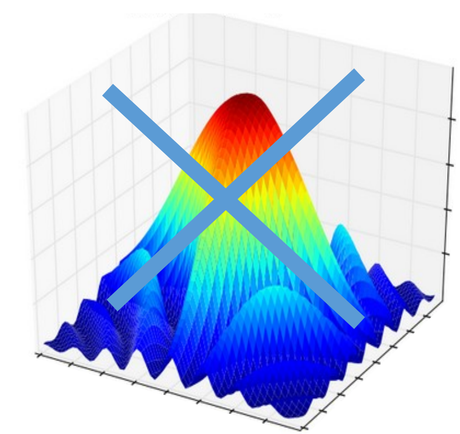
There is no emerging quantity

What should be avoided :
Define a temperature
=> entropy
=> back to thermodynamics

Conclusion on thermostatics

- Economics do not have the structure of thermodynamics
- No general optimization (but locally)
- No fundamental equation // variational principle
- No ergodicity
- No (?) emerging properties (as T, P...)
- No hyper surface of utilities

- And economic systems are not
  in equilibrium !



Schumpeter, Onsager and Prigogine

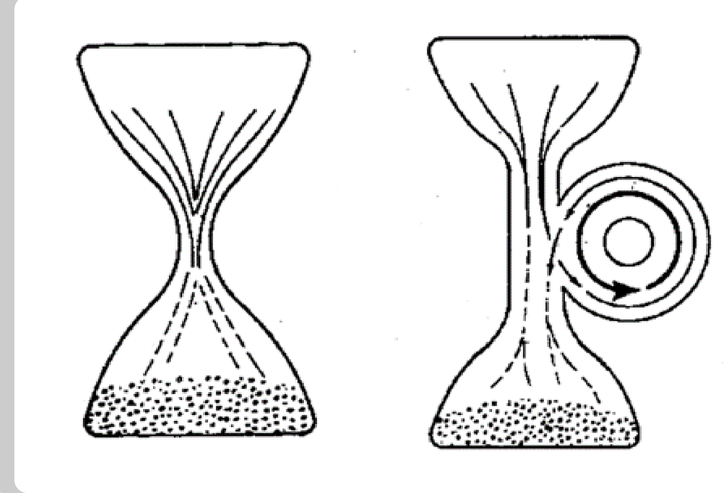
Not mechanics

"Walras arrived at his unique equilibrium by starting from a "*prix crié par hasard*" and allowing people to say what quantities they would be willing to demand and to supply at that price without actually buying or selling until that initial price is *"par tâtonnement"* so adjusted as to equate quantity supplied and quantity demanded...**The presence of friction, will, of course, always entail an equilibrium different from that which would otherwise be reached**, as well as slow up progress toward equilibrium..."(Schumpeter 1939)

Not at equilibrium

"The first puts into its proper light our former statement, that disturbances of equilibrium arising from innovation **cannot be currently and smoothly absorbed**. In fact, it is now easy to realize that those disturbances must necessarily be "big," in the sense that they will disrupt the existing system and enforce a distinct process of adaptation. **This is independent either of the size of the innovating firm** or of the importance of the immediate effects their action would in itself entail." (Schumpeter 1939)

Roegen, quality : « matter matters too »



The far out of equilibrium assumption is ok,
but
there is no need of a fourth thermodynamics principle for matter.

"The idea that the economic process is not a mechanical analogue, but an entropic, unidirectional transformation began to turn over in my mind long ago, as I witnessed the oil wells of the Ploesti field of both World Wars' fame becoming dry one by one and as I grew aware of the Romanian peasant's struggle against the deterioration of their farming soil by continuous use and by rains as well. **However, it was the new representation of a process that enabled me to crystallize my thoughts in describing for the first time the economic process as the entropic transformation of valuable natural resources (low entropy) into valueless waste (high entropy).**" (Roegen 1971)

An alternative route to an ecological modelling

- Physical cell build on energy conversion formalism
  - Integration of a limited quantity of primary resource
  - Intensity and friction
  - Quality and quantity
  - Inclusion of natural and forced recycling
- Stock flow consistent model
- Classical economic framework (Goodwin)

How to ?

Give up the full thermodynamic paradigm (Integrability, Equation of state…)

Conservative laws (no full substitutability)

Non conservation of some physical quantities (utility...)

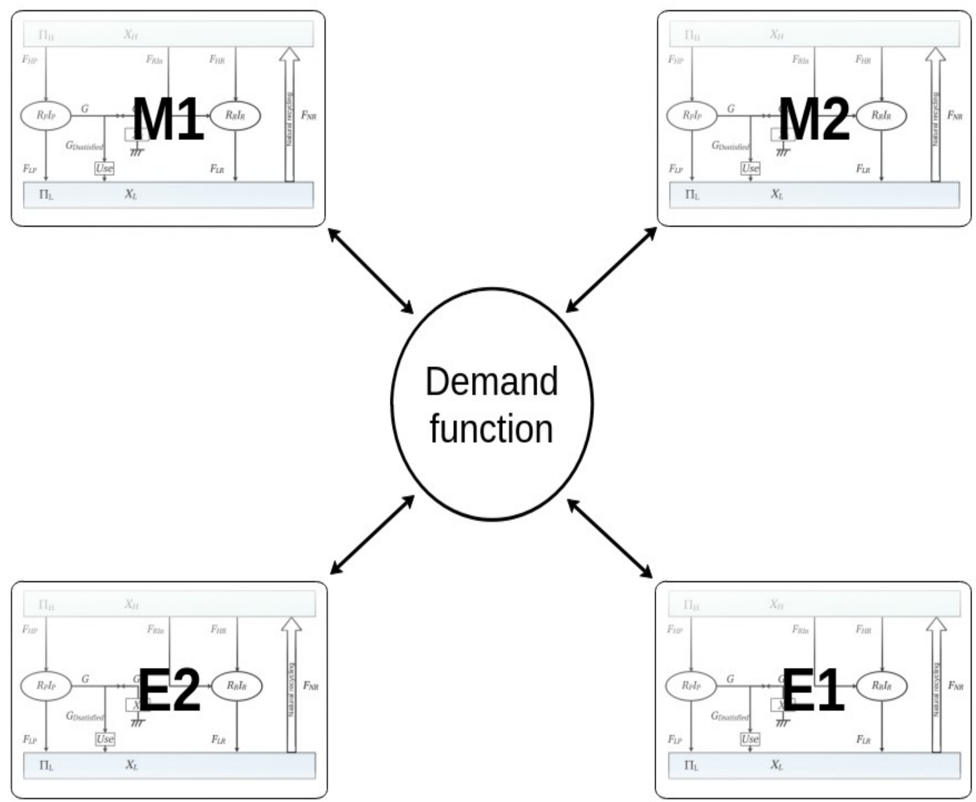
Difference between extensive and intensive physical quantities

Intensive physical quantities are the potentials from which we can derive forces

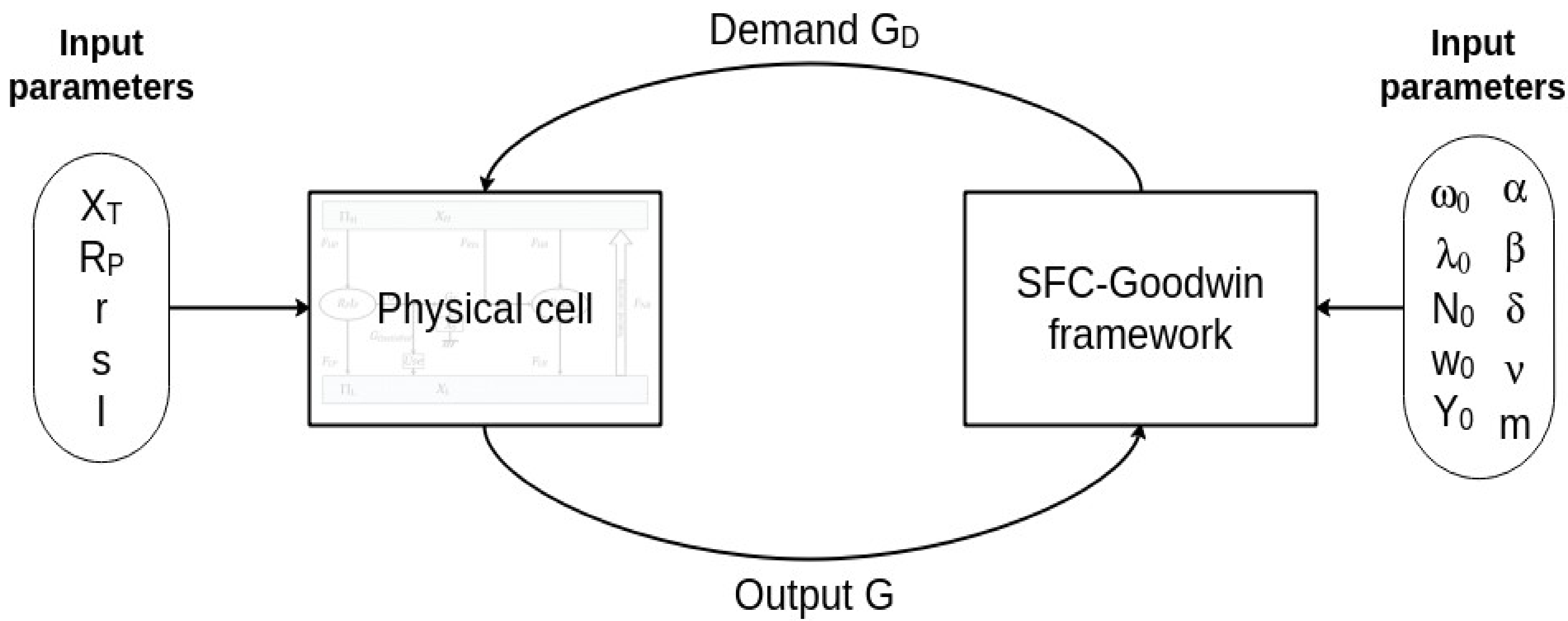
There is no economic entropy but we can consider quantity and quality approach

Time does matter BUT, there is no variational principle

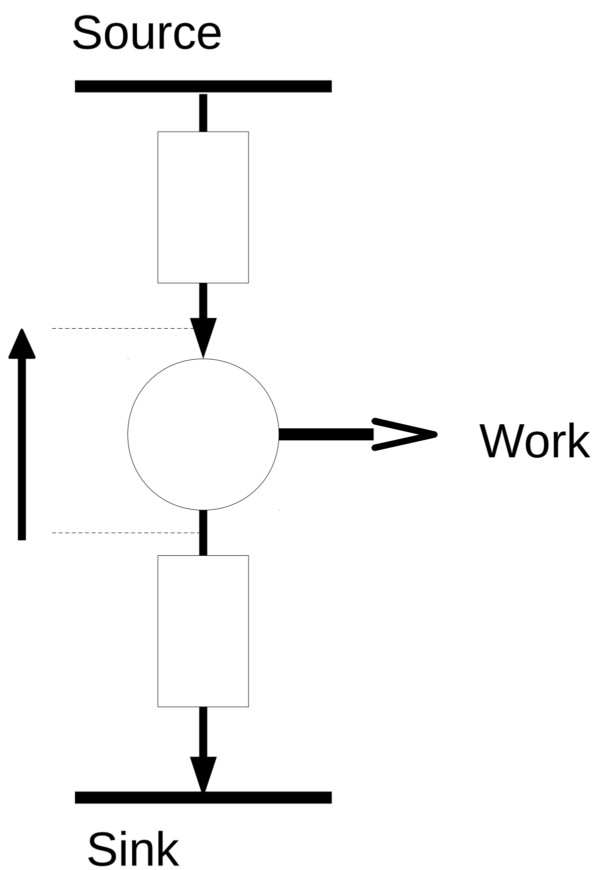
Physical Cells with four sector interconnected



Physical cell into a SFC-Goodwin Framework

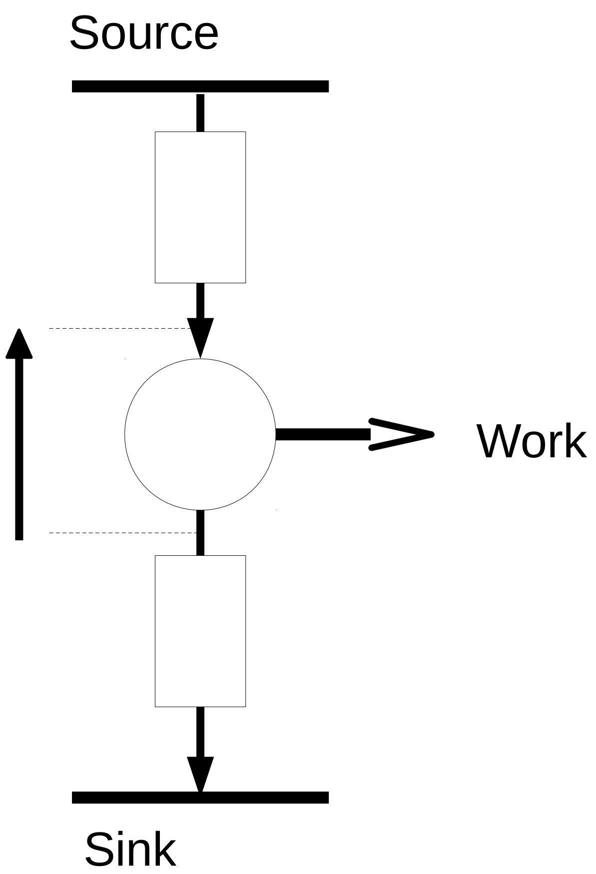


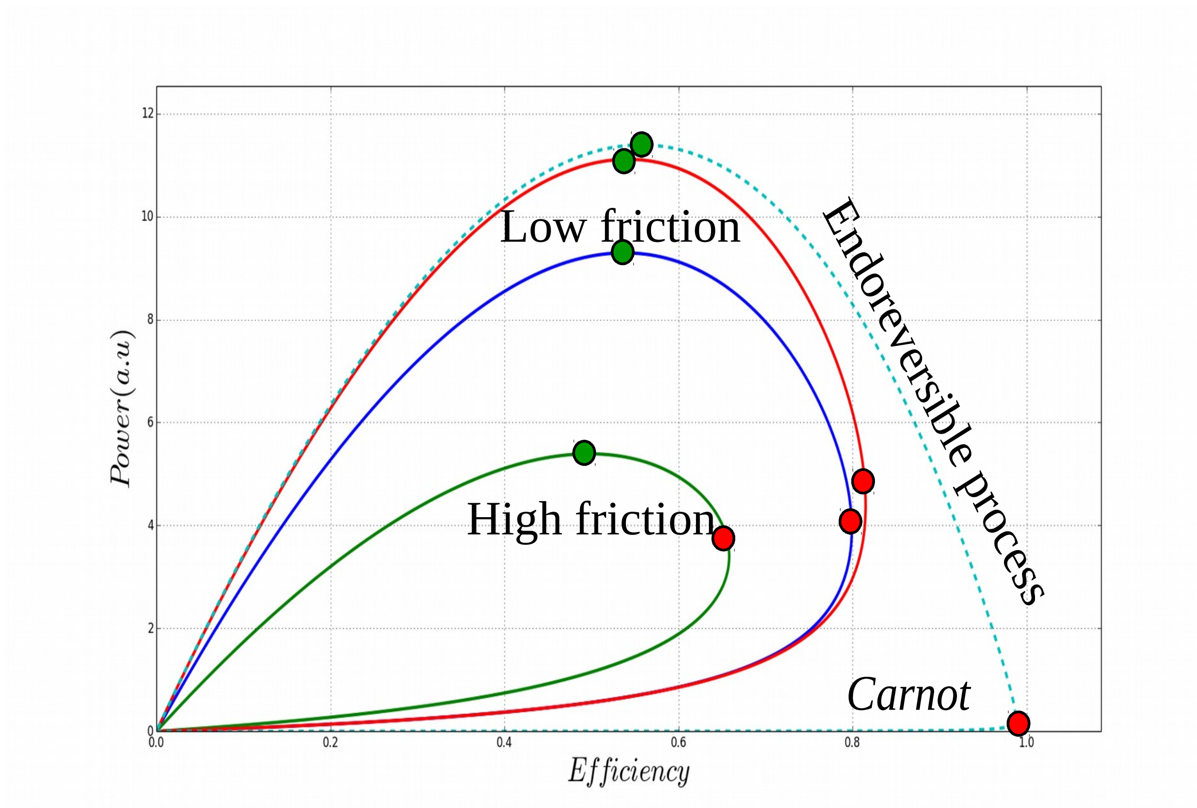
Energy conversion engine



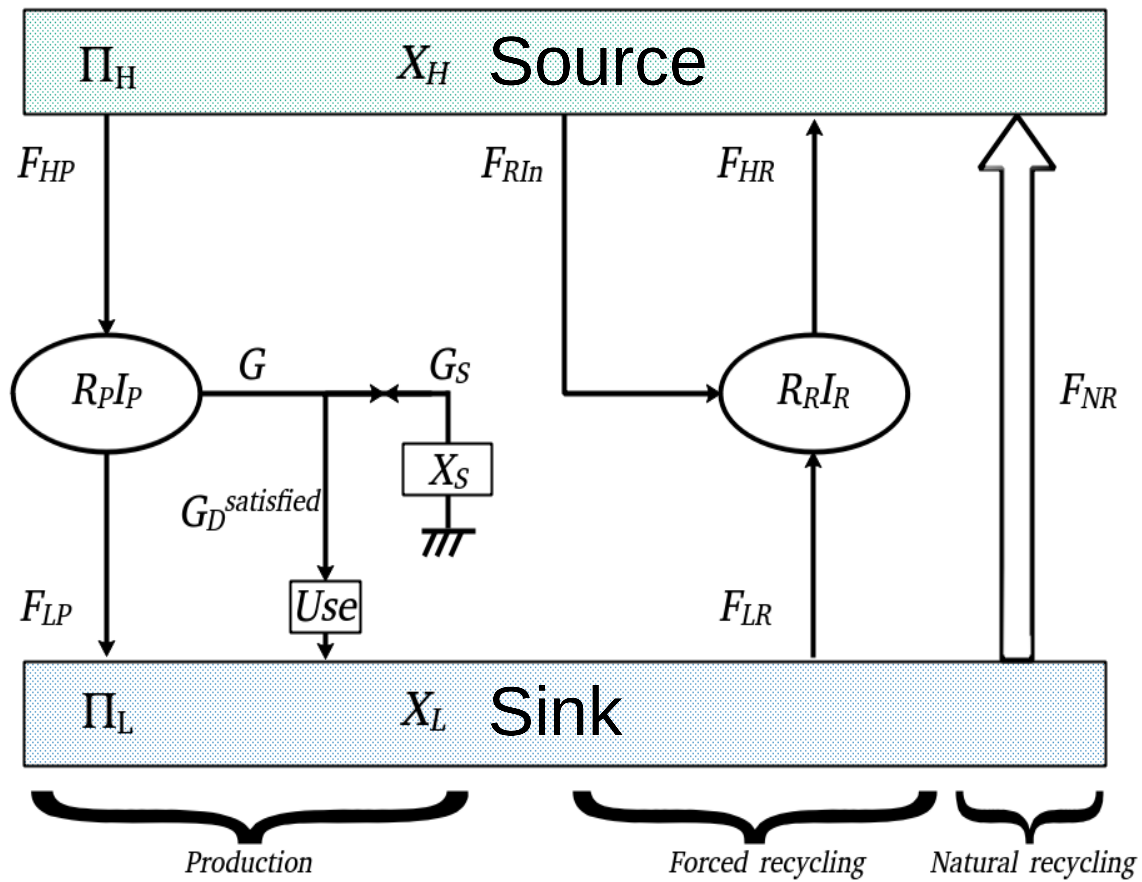
- Economic system as a living organism which metabolises E&M
- Flow and stock economies (Wrigley, Astrid Kander)
- Thermodynamic constraints on economy
  - Inputs availability
  - Output sink capacity
  - Working intensity

Energy conversion engine



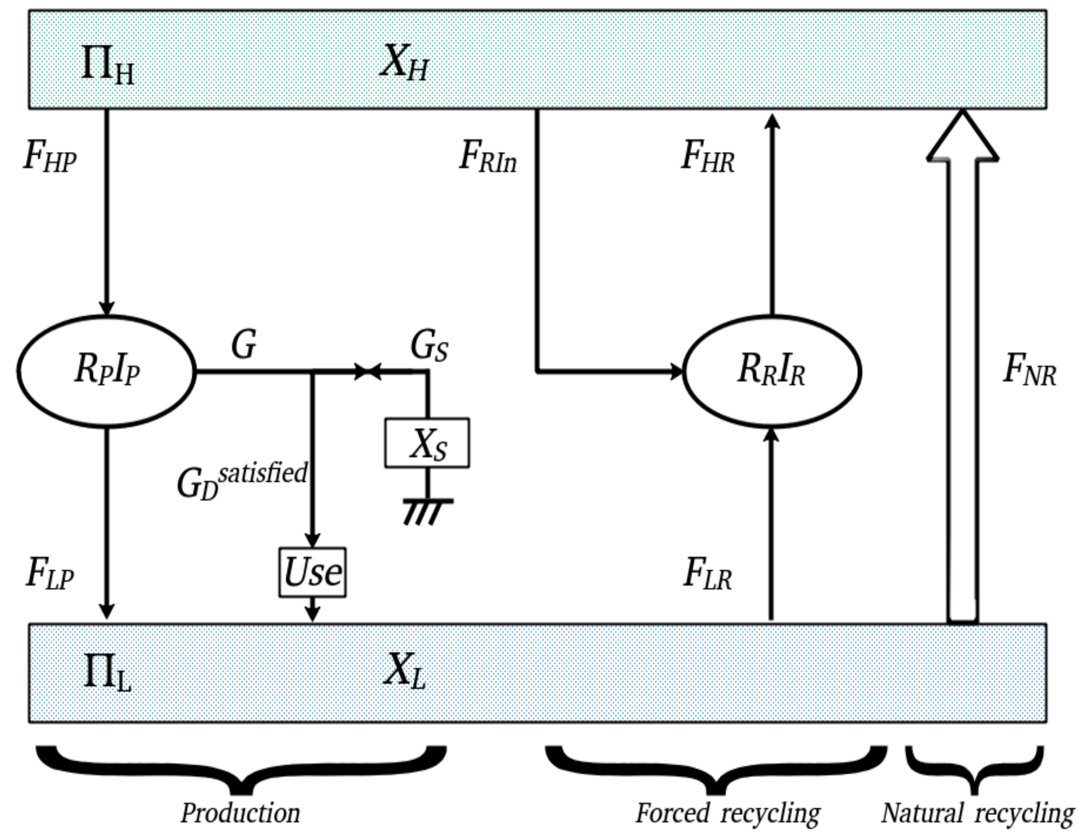


Global scheme of resource economy



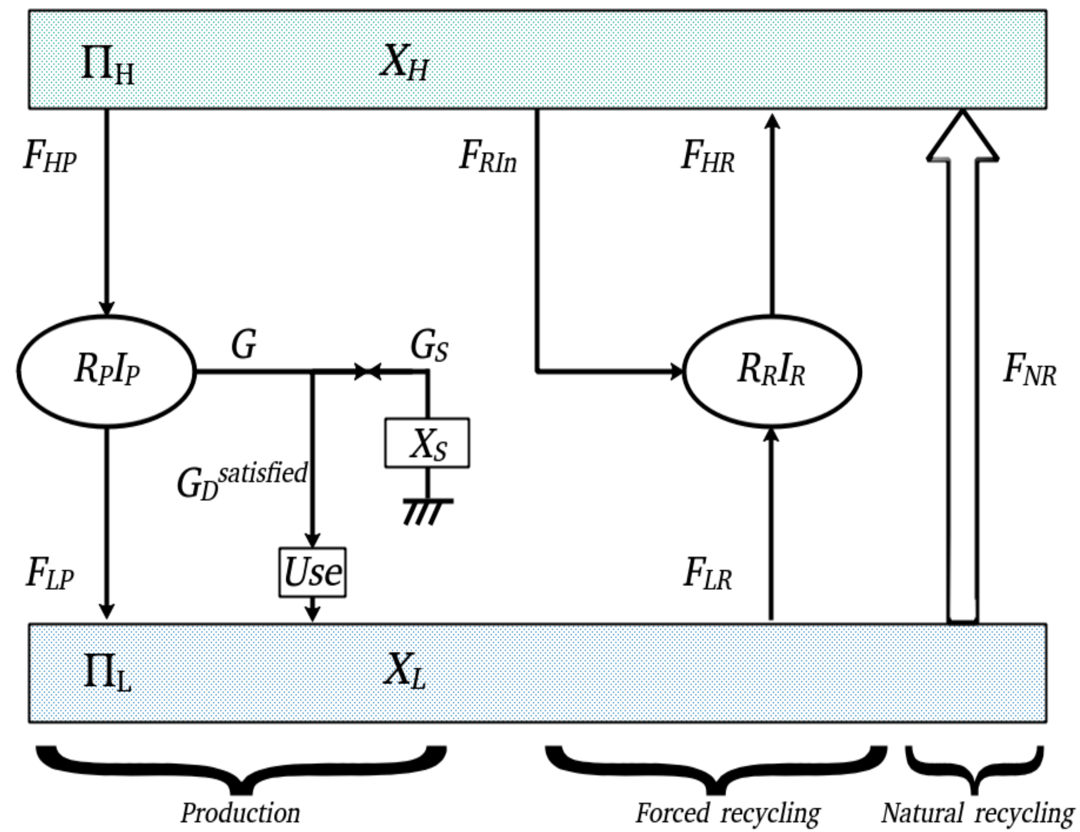
Balance equations: production zone

$$\begin{aligned}
X_T &= X_H + X_L + X_S \\
I_P &= \beta_P I \\
F_{HP} &= \Pi_H I_P \\
F_{LP} &= \Pi_L I_P + R_P I_P{}^2 \\
G &= F_{HP} - F_{LP} \\
G &= G_D^{satisfied} + G_S
\end{aligned}$$



Balance equations: recycling zone

$$
\begin{aligned}
I_R &= \beta_R I \\
F_{HR} &= \Pi_H I_R \\
F_{LR} &= \Pi_L I_R - R_R {I_R}^2 \\
F_{RIn} &= F_{HR} - F_{LR}
\end{aligned}
$$



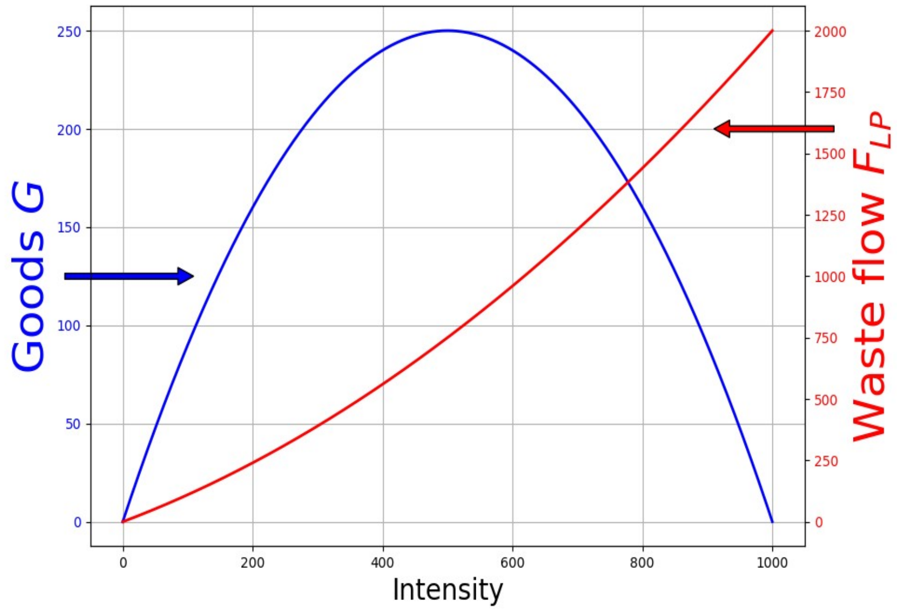
Case studies

- I is chosen in order to satisfy G

$$-R_P I^2 + \Delta\Pi I - G_D = 0$$

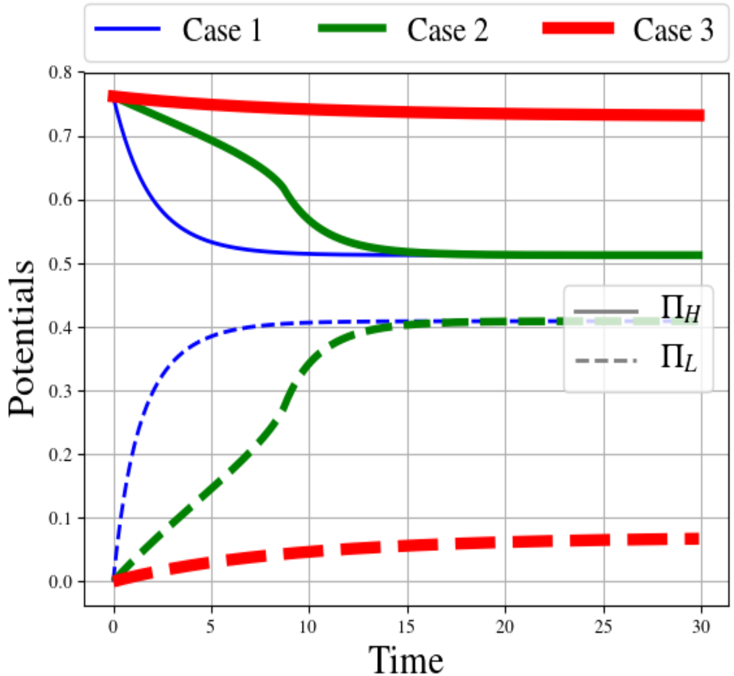
- Fixed recycling intensity
- Optimum intensity satisfies the demand with minimum waste
- Maximum quantity of goods possible to produce

$$I_P^{max} = \frac{\Delta\Pi}{2R_P}$$



Intensity impact

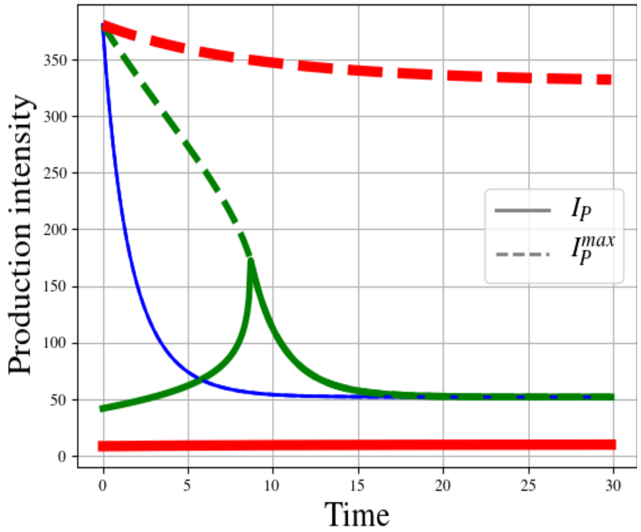
- 1/ Max intensity
- 2/ Optimal intensity
  - Corresponds to the exact demand
- 3/ 20% of the optimal intensity
  - Weak intensity

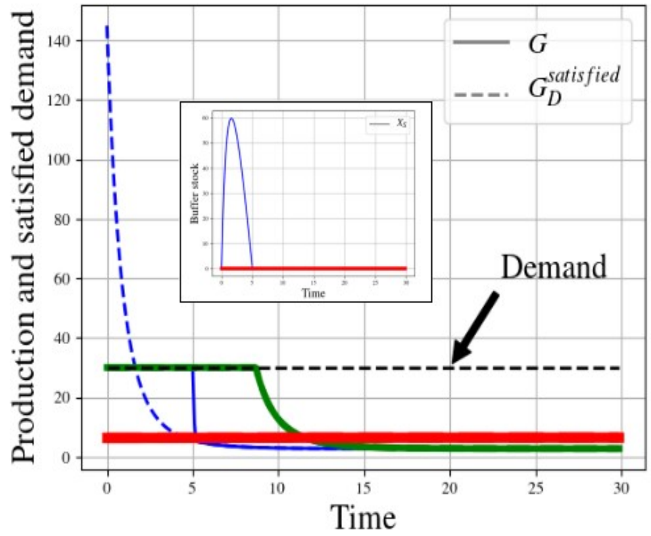


- Max recycling intensity

$$I_R = I_R^{max} = \frac{\Pi_L}{2R_R}$$

Intensity impact

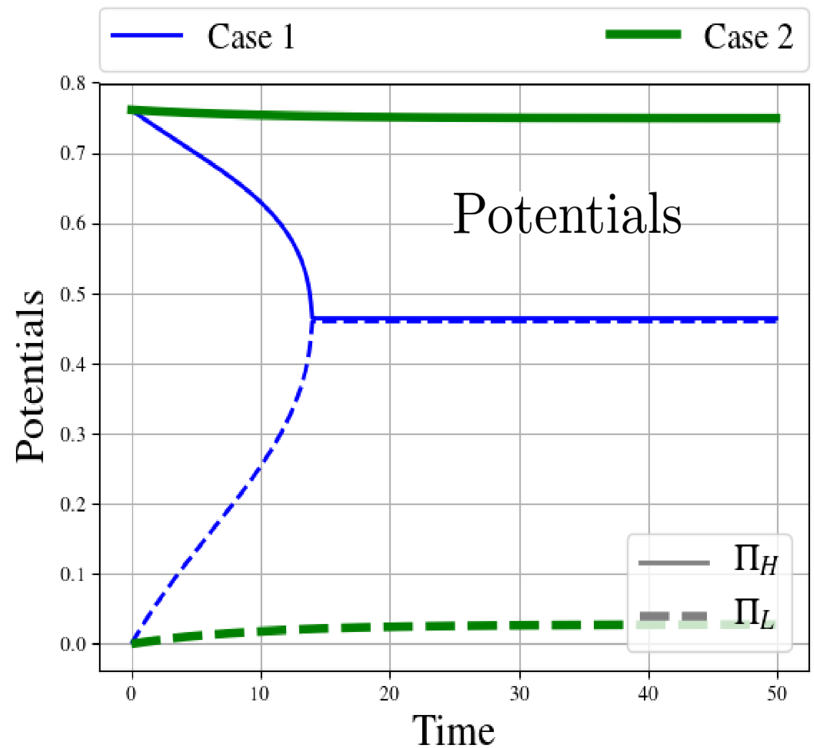




- Max intensity:
  - Buffer effect
- Optimal intensity
  - production satisfies the demand and then drop
- Weak intensity
  - Demand is never satisfied but never collapse.

Friction Impact

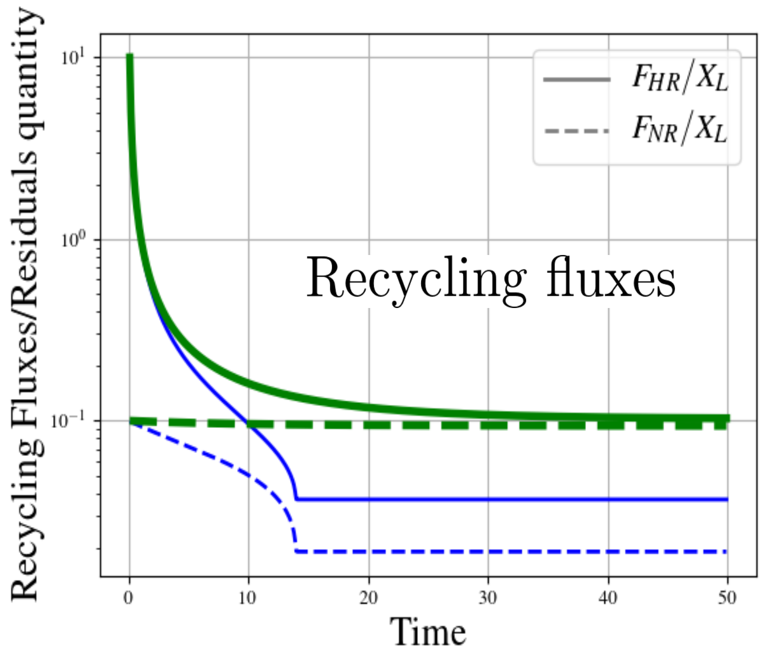
- Production or recycling friction can vary
- 1/ low friction
  - $R_p = 10^{-5}$
  - Good (efficient) production system
- 2/ high friction
  - $R_p = 10^{-1}$
  - Poor production system

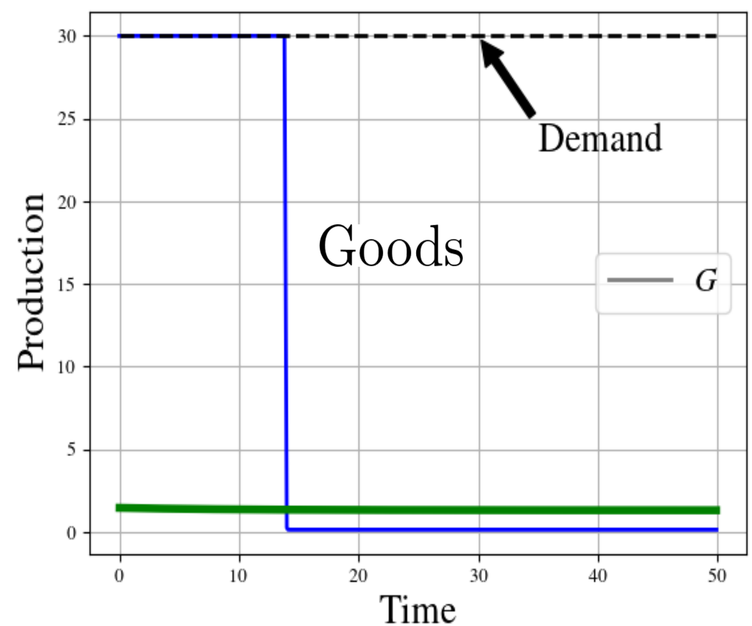


Friction Impact

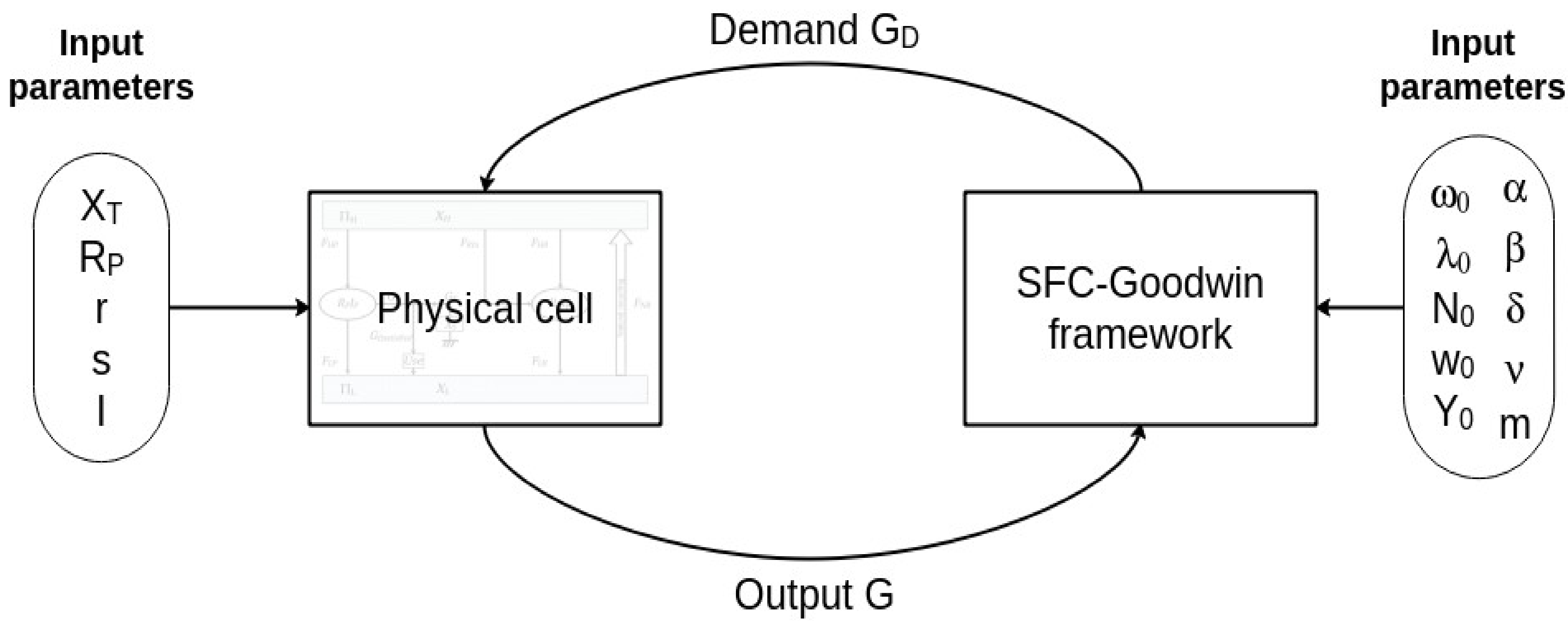
- 1/ low friction
- Capable of attaining the demand
- High consumption and waste
- Potentials finally are pinched

- 2/ high friction
- Production is difficult
- Low consumption and waste
- Potential weakly pinch

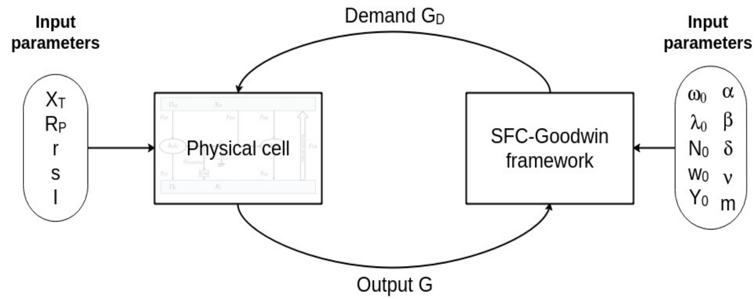




Physical cell into a SFC-Goodwin Framework



Physical cell into a SFC-Goodwin Framework



- Resource is a global resource
- Starting from Y_0 the initial demand

$$G_D = Y_0$$

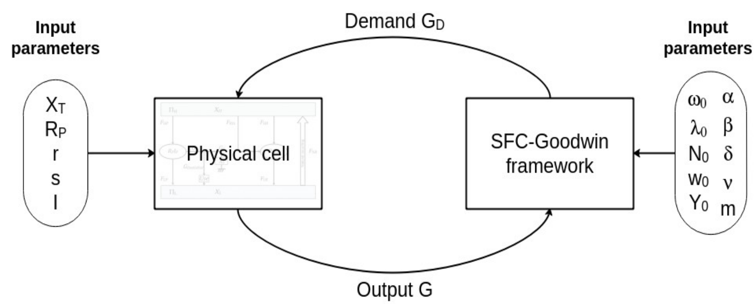
- The physical cell takes into account the demand and depend on resource, friction and recycling

- We obtain G_D
- We then try to reach $G=G_D$ by modifying *I* in

$$-R_P I^2 + \Delta\Pi I - G_D = 0$$

- Hence Intensity depends on G_D, R_P, and $\Delta\pi$

Balance equations: discrete stock balance equations



- Two sectors
  - Households
  - Firms

$$S^h = -pC + W$$

0 = consumption - wage

$$Y = C + I$$

Output = consumption – investment

$$\Pi = pY - wL$$

Profit = Output - Wage

- No bank
- No saving / debt
- No government sector

Goodwin framework

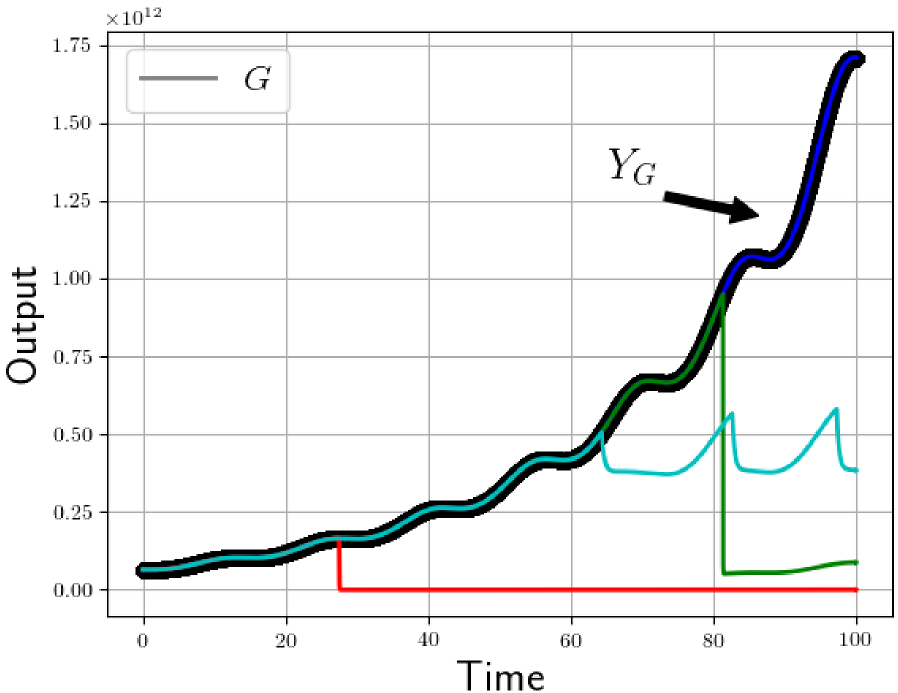
- Prey predator model
  - Wageshare
  - Employment rate
  - Wage
  - Labour
  - Capital
  - Investment
  - Prices

$$
\begin{aligned}
\dot{\omega}_G &= \omega_G \, [\phi(\lambda_G) - \alpha] \\
\dot{\lambda}_G &= \lambda_G \left[\frac{1-\omega_G}{\nu} - \alpha - \beta - \delta \right] \\
\dot{w}_G &= w_G \, \phi(\lambda_G) \\
L_G &= \lambda_G N_G \\
\dot{K}_G &= Inv_G - \delta K_G \\
Inv_G &= Y_G \, (1 - \omega_G) \\
p_G &= (1+m) \, \frac{w_G L_G}{Y}
\end{aligned}
$$

5 production scenarios

Scenario	**G**	1	2	3	4
X_T	-	$\mathbf{10^8}$	100.0	100.0	100.0
R_P	-	0.001	0.001	0.001	**0.1**
Features	Goodwin pathway	Infinite resources	Standard example	No Forced recycling	High friction

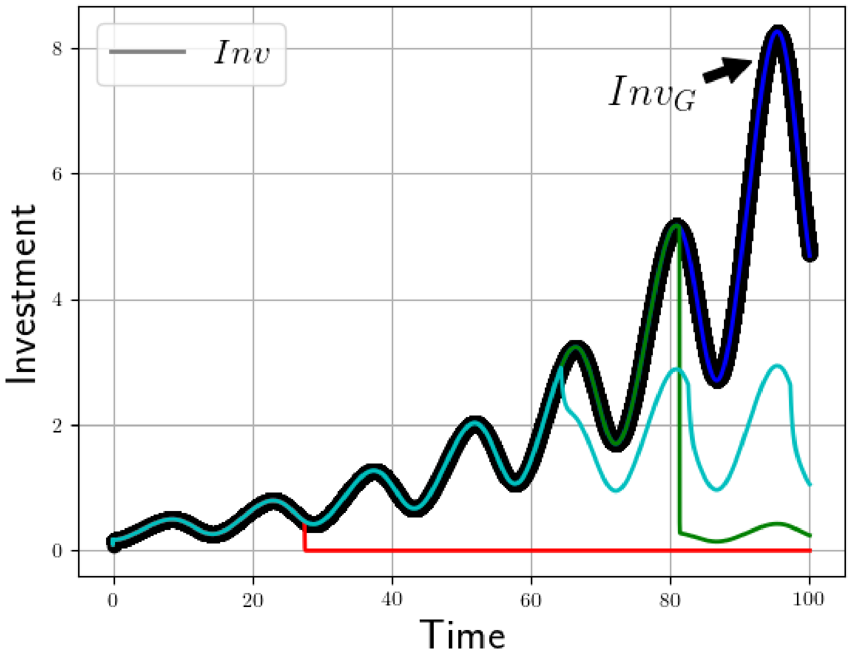
Production of output



- Goodwin = infinite resources
- Finite resources => collapse
- Natural recycling => early collapse + low output
- High friction => output saturation then oscillation (due to capital dependency of Imax) higher than with lower Rp !

Scenario	**G**	1	2	3	4
Features	Goodwin pathway	Infinite resources	Standard example	No Forced recycling	High friction

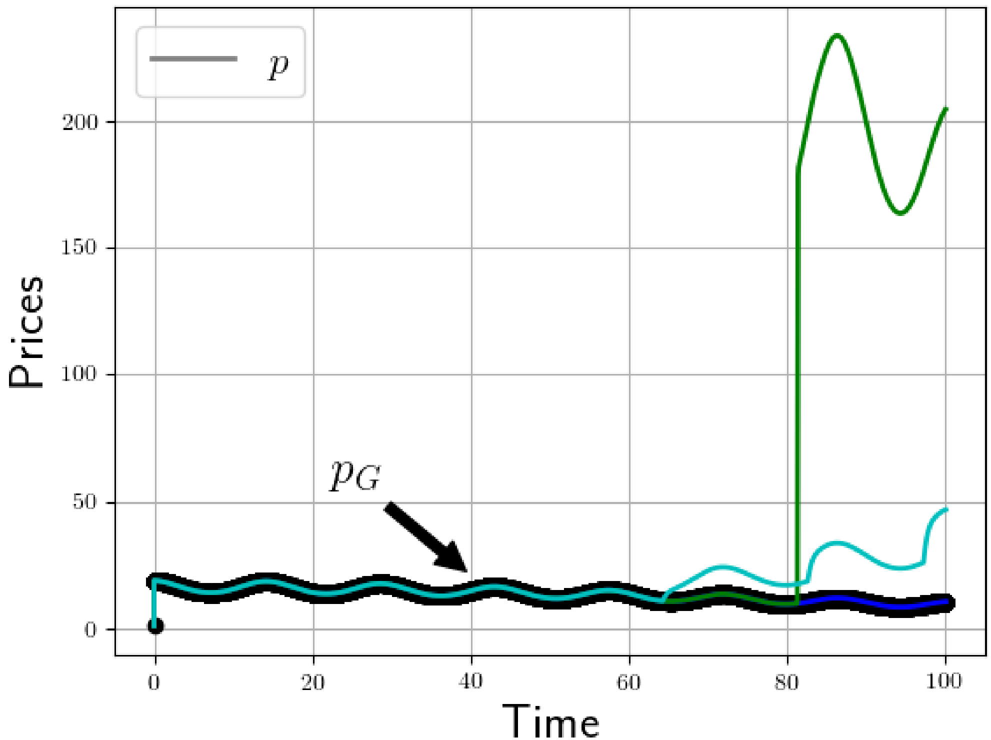
Investment



Investment follow the output trend !
no investment => system degrades for 2 and 3

Scenario	G	1	2	3	4
Features	Goodwin pathway	Infinite resources	Standard example	No Forced recycling	High friction

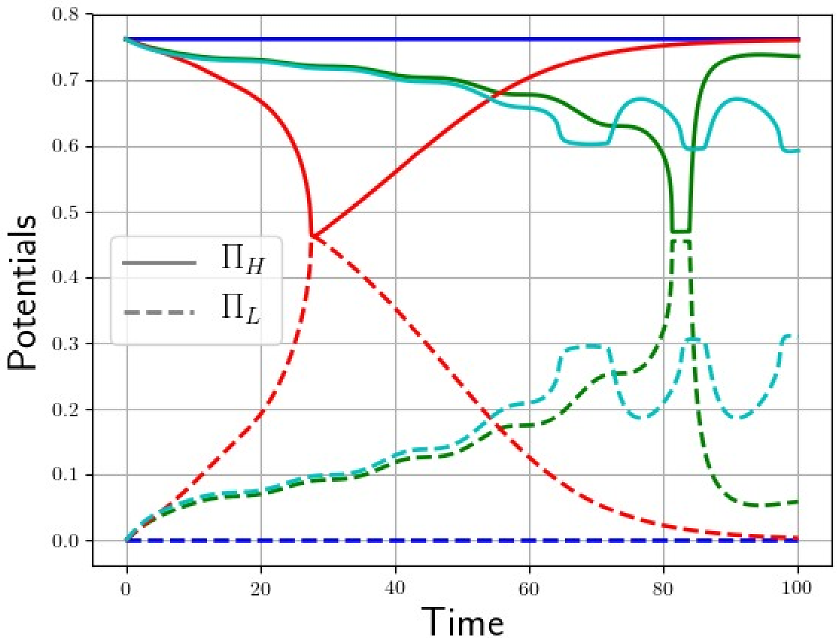
Prices



Prices follow the output trend !

Scenario	**G**	1	2	3	4
Features	Goodwin pathway	Infinite resources	Standard example	No Forced recycling	High friction

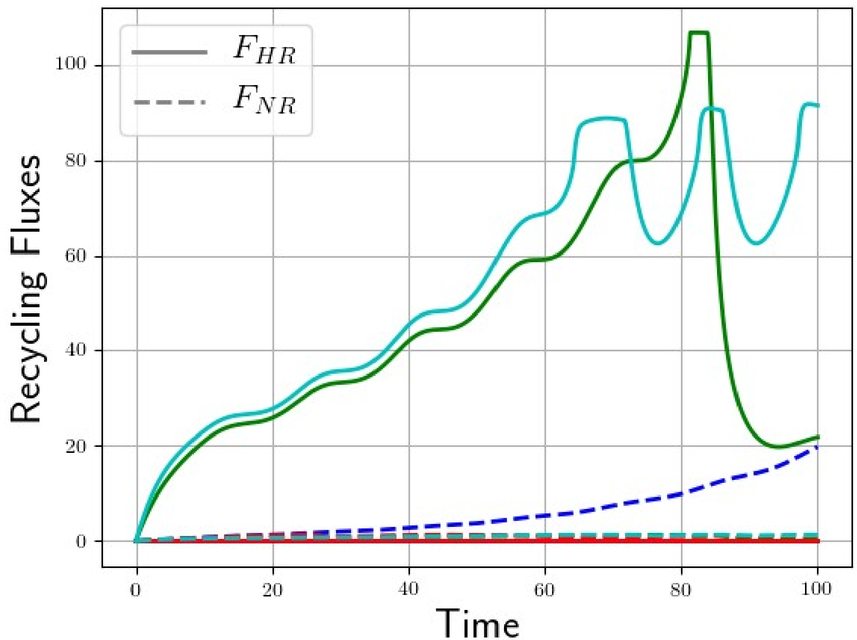
Potentials



- Infinite resources => constant potentials
- Finite resources => potentials decrease (production collapse) and then increase (sync with decrease in demand)
- 3/ Rapid exhaustion of potentials followed by slow increase (due to friction)
- 4/ Conserved potential difference after maximum production is reached

Scenario	**G**	1	2	3	4
Features	Goodwin pathway	Infinite resources	Standard example	No Forced recycling	High friction

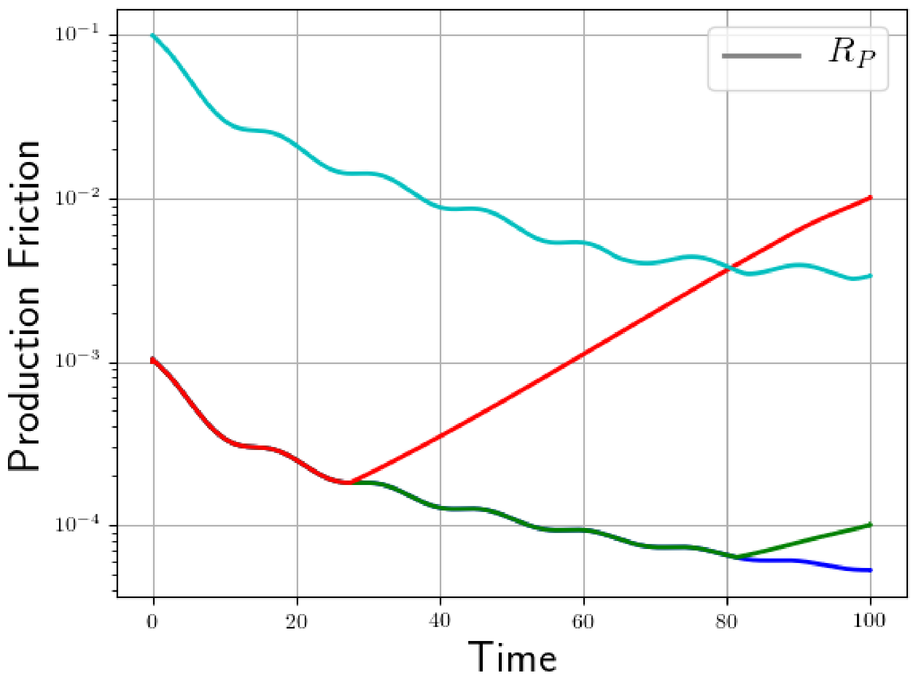
Recycling fluxes



- Infinite resources => no forced recycling (due to the low potential)
- Finite resources => strong forced recycling until potential collapse.
- High friction => strong forced recycling due to conserved potential difference

Scenario	**G**	1	2	3	4
Features	Goodwin pathway	Infinite resources	Standard example	No Forced recycling	High friction

Production friction



- Infinite resources and high friction => monotonic decrease of friction
- 2 and 3 Finite resources => friction decrease until potential collapse then increase, system degrades !

Scenario	**G**	1	2	3	4
Features	Goodwin pathway	Infinite resources	Standard example	No Forced recycling	High friction

Conclusion

- E&M are slightly taken into account in most economic model
- We propose a ecological modeling composed of
  - Thermodynamics categories (quality/quantity)
  - Physical cells (using intensity, friction categories)
  - Stock Flow Consistent model
  - Goodwin (prey predator) economic framework
- We regain reasonable behavior of finite size system
- Next step is the calibration of the model

Balance equation flux / potentials

$$
\begin{aligned}
\frac{\Delta X_H}{\Delta t} &= F_{NR} - F_{HP} + F_{HR} - F_{RIn} \\
\frac{\Delta X_L}{\Delta t} &= -F_{NR} + F_{LP} - F_{LR} + G_D^{satisfied} \\
\Pi_H &= \tanh\left(\alpha \frac{X_H}{X_T}\right) \\
\Pi_L &= \tanh\left(\alpha \frac{X_L}{X_T}\right)
\end{aligned}
$$

Discrete system

$$
\begin{aligned}
\omega_t &= \omega_{t-1}\left[1+\Delta t\left(\phi(\lambda_{t-1})-\alpha\right)\right] \\
\lambda_t &= \lambda_{t-1}\left[1+\Delta t\ \left(\frac{1-\omega_{t-1}}{\nu}-\alpha-\beta-\delta\right)\right] \\
w_t &= w_{t-1}\left[1+\Delta t\left(\phi_0+\phi_1\lambda_{t-1}\right)\right] \\
L_t &= \lambda_t N_t \\
K_t &= K_{t-1}\left(1-\Delta t\delta\right)+\Delta t\ Inv_t \\
Inv_t &= G_t\left(1-\omega\right) \\
\Pi_t &= p_t G_t - w_t L_t \\
p_t &= (1+m)\,\frac{w_t L_t}{G_t} \\
G_{Dt+1} &= G_t\ \left[1+\Delta t\ \left(\frac{1-\omega_t}{\nu}-\delta\right)\right] \\
R_{Pt+1} &= \frac{R_{P0}K_0}{K_t}+4.0\ 10^{-5}
\end{aligned}
$$